

Markups: Risk and Market Power*

Chun-Beng Leow

Princeton

October 26, 2026

Abstract

I quantify the relative contributions of oligopoly market power and idiosyncratic risk to firm-level markups. I develop a general equilibrium model of granular firms competing in a product network under cost and demand uncertainty, where managers face source-dependent risk penalties. Exploiting a latent factor structure in revenues and profits, I jointly estimate baseline market power, and the risk markups for U.S. public firms. I find that idiosyncratic risk accounts for only 4% of aggregate markups, suggesting that it is not the primary driver of markups. Antitrust and competition regulation should remain central to addressing markups.

*I am indebted to my advisor Jonathan Payne for his support and guidance throughout the project. I thank Mark Aguiar, John Grigsby, Magnus Irie, Nobuhiro Kiyotaki, Moritz Lenel, Ernest Liu, Aleksei Osolkov, Richard Rogerson, Karthik Sastry, Motohiro Yogo, and other seminar participants for their thoughtful comments.

1 Introduction

The macroeconomic literature has documented substantial aggregate markups in the U.S. economy, with estimates ranging from 20% to 60% and considerable heterogeneity across firms (De Loecker et al., 2020; Traina, 2018; Benkard et al., 2025). However, the sources of markups remain debated. I focus on two explanations, a traditional industrial organization view that attributes markups to market power and a more recent view that highlights compensation for bearing uninsurable idiosyncratic risk. The first explanation emphasizes heterogeneity in market power arising from product-market network competition, while abstracting from risk (Pellegrino, 2025). The second focuses on how uninsurable idiosyncratic risk distorts firms’ decisions and markups, but abstracts from the rich market structure in which firms operate (Boar et al., 2026). Distinguishing between these explanations matters because they have different policy implications. If markups mainly reflect market power, the policy focus should remain on competition policy and antitrust enforcement. However, if markups reflect compensation for bearing undiversifiable risk, policies that improve risk sharing or align firms’ incentives with those of the social planner may be more effective. This paper examines how much of observed markups reflects market power versus compensation for idiosyncratic risk, and the welfare implications of each mechanism.

I make three main contributions. First, I develop a general equilibrium model of firms competing in product-market networks under uncertainty. I characterize how uninsurable idiosyncratic risks affect firms’ pricing decisions and provide a decomposition of markups into components attributable to market power, risk premium, and the interaction between product-market networks and risk. Second, I develop an estimation method that combines data on firms’ product-market networks from Text-Based Network Industry Classifications (TNIC) with financial data. I use a latent factor structure to identify firm-level idiosyncratic risk and quantify the contributions of market power and risk to observed markups. Third, applying my framework to U.S. public firms from 1990 to 2020, I find that idiosyncratic risk accounts for only 4.10% of aggregate markups, suggesting that it is not the dominant source of aggregate markups.

To study the roles of product-market competition and risk in shaping markups, I extend the network oligopoly model of Pellegrino (2025). This framework captures firms’ competitive relationships through a product network, allowing demand elasticities to vary across firms and cross-price effects to depend on firms’ product-market relationships. I extend the model to incorporate both production and demand uncertainty, with firm-specific volatilities capturing heterogeneity in firms’ exposure to each source of risk. Firms have mean-variance preferences over profits, and I allow the penalty on profit variance to differ between cost and

demand risk, capturing differences in firms' responses to the two sources of uncertainty.

The model characterizes how different sources of risk affect firms' pricing decisions and decomposes the resulting markups into a *cost risk premium*, a *demand risk premium*, and *risk market power*. Under Bertrand competition, I show that a firm's pricing response depends on the source of uncertainty. Firms facing higher cost volatility raise prices to lower expected demand and limit total cost volatility. Conversely, higher demand uncertainty induces firms to lower prices to compress profit margins and mechanically reduce profit volatility. These responses propagate through the product network. When competitors face cost uncertainty and increase prices, the focal firm experiences an outward demand shift and raises its own price.

Estimating the model is challenging because firm-level prices are not directly observed in U.S. public-firm data, making it difficult to identify how idiosyncratic risk affects pricing decisions. Existing approaches either abstract from risk in models of product-market competition (Pellegrino, 2025) or use macroeconomic models with simplified market structures and homogeneous risk distributions that target aggregate moments (Boar et al., 2026). I address this challenge by exploiting the time-series variation in firms' financial data. I model firms' fundamentals using a latent factor structure, which implies corresponding factor model for revenues and profits. This allows me to identify the latent factors and estimate firm-level cost and demand risk variances within the structural model. I combine this approach with TNIC data to identify the contribution of product-market competition to observed markups. Since the data do not separately identify all firm-level risk parameters, I impose semiparametric restrictions that allow factor loadings and risk penalties to vary with observable firm characteristics. I assess the robustness of my estimates to alternative specifications and groupings of these characteristics.

I estimate the model using data on U.S. public firms from 1990 to 2020 and find that oligopoly competition accounts for 95.90% of the revenue-weighted average markup. Among the risk-induced channels, the cost risk premium contributes 4.53%, while the demand risk premium reduces the average markup by 0.87% and the strategic propagation of these forces through risk market power contributes 0.44%. Furthermore, I document significant cross-sectional heterogeneity: smaller firms exhibit a more negative demand risk premium and are more sensitive to the equilibrium effects of risk market power. Finally, because my estimates are derived from the reduced-form firm problem, they are robust to the microfoundations of the risk penalties.

To evaluate the welfare implications, I microfound the reduced-form, source-dependent risk penalties through a principal-agent framework. The principal offers the firm manager a linear contract that ties compensation to both expected and realized revenues and profits.

Because the manager exerts hidden costly effort to manage risk, the principal must pay an agency premium to induce mitigation. To isolate this friction, I restrict the principal from using the contract as a strategic tool to alter oligopoly market power. In this model, the resulting level of risk management is lower than the first best.

I utilize the principal-agent framework to bound the welfare losses in my economy. Total deadweight loss amounts to 16.08% of the first-best surplus. Further decomposing this inefficiency, I show that oligopoly competition accounts for the vast majority (91.22%) of the total deadweight loss. Risk frictions account for the remainder, with cost and demand risk contributing 8.67% and 0.11%, respectively. Notably, while market power dominates the aggregate deadweight loss, the welfare distortion induced by risk remains economically meaningful. Mitigating risk management efforts to the first-best level yields a 1.39% gain in total surplus.

I contrast my findings with Boar et al. (2026), who estimate that risk drives the majority of markups among private firms. I trace this divergence to a specific structural vulnerability in their framework alongside fundamental differences in the data. To match aggregate output innovations, their model imposes a homogeneous, fat-tailed distribution that inflates the covariance between the stochastic discount factor and production shocks, mechanically driving up the implied markup. I show that simply allowing for cross-sectional heterogeneity with standard Gaussian shocks matches the aggregate data while avoiding this mechanical amplification of the risk channel. Although estimating my framework on a private firm sample would perfectly isolate the role of these structural choices, performing such a direct comparison is empirically infeasible. My reliance on U.S. public firms is strictly dictated by data availability, as the granular product network data required to jointly estimate market power and idiosyncratic risk does not exist for private firms.

Related Literature. I contribute to the extensive literature investigating the determinants of markups (Grullon et al., 2019; Autor et al., 2020; David et al., 2022; Kwon et al., 2024; Burstein et al., 2025). More specifically, my work is closely related to recent studies showing that uninsurable risk generates wedges between prices and marginal costs (Boar et al., 2026; Di Tella et al., 2025; Bhandari et al., 2026). While these models emphasize how risk distorts input choices before productivity is realized, my mechanism, alongside recent work by Hasenzagl and Pérez (2026) and Eeckhout and Veldkamp (2024), operates directly through the pricing decision. Hasenzagl and Pérez (2026) show that risk-averse firms post prices before realizing costs and demand to actively manage risk exposure, where cost uncertainty induces firms to raise markups and demand uncertainty prompts them to lower markups.

Methodologically, I extend the granular network oligopoly framework of Pellegrino (2025). Their characteristic-based demand system utilizes textual product descriptions (Hoberg and Phillips, 2016) to map a continuous product space and estimate cross-price elasticities of substitution among competing firms.¹ Building upon this, I develop an estimation strategy that allows me to identify parameters governing firm-level idiosyncratic risk by leveraging the time series dimension of financial statement data.

Finally, I contribute to the literature on the welfare costs of markups (Zhelobodko et al., 2012; Behrens et al., 2020; Edmond et al., 2023). I enrich this work by incorporating rich market competition and introducing a heterogeneous distribution of uninsurable idiosyncratic risk into the welfare analysis (Boar et al., 2026).

2 Descriptive Evidence

This section presents reduced-form evidence on the relationship between markups and idiosyncratic risk. I document a positive correlation between firms’ idiosyncratic risk and markups, suggesting that risk may contribute to observed markup variation. Furthermore, the relationship varies across risk proxies, pointing to differences in how distinct sources of risk relate to markups. This motivates the model developed in the next section, which distinguishes between different sources of risk and their effects on markups.

I estimate markups for publicly traded US firms over the period 1990–2020 using the production-function approach of De Loecker et al. (2020).² I implement the estimation using code provided by Basile Grassi, which reproduces the markup estimation procedure in De Loecker et al. (2020).³ The required financial data are obtained from Compustat. Following the literature, I winsorize the data at the 1st and 99th percentiles.

I construct two proxies for idiosyncratic risk using firms’ cost of goods sold (COGS) and revenue (REV). Let $\overline{COGS}_i$ and $\overline{REV}_i$ denote their respective firm-level means. To purge aggregate shocks and firm-specific growth, I perform the following regression for each of the variables

$$\begin{aligned}\frac{COGS_{it}}{\overline{COGS}_i} &= \alpha_i^c + \delta_{s(i)t}^c + \beta_i^c t + u_{it}^c, \\ \frac{REV_{it}}{\overline{REV}_i} &= \alpha_i^r + \delta_{s(i)t}^r + \beta_i^r t + u_{it}^r,\end{aligned}$$

¹This approach extends the foundational literature on hedonic demand (see Lancaster, 1966; Rosen, 1974; Epple, 1987).

²The sample period is chosen to align with the subsequent analysis, which requires product-network data.

³The Python code is available at <https://sites.google.com/site/grassibasile/home/code/markup-and-production-function-estimation>.

where α_i^c and α_i^r denote firm fixed effects, β_i^c and β_i^r capture firm-specific linear growth, and $\delta_{s(i)t}^c$ and $\delta_{s(i)t}^r$ denote sector by time fixed effects. The residuals, $\hat{u}_{it}^c$ and $\hat{u}_{it}^r$, proxy idiosyncratic cost and revenue shocks, respectively. I define firm-level cost and revenue volatility as the within-firm variances of the residuals

$$\hat{\sigma}_{cogs,i}^2 = \text{Var}(\hat{u}_{it}^c), \quad \hat{\sigma}_{rev,i}^2 = \text{Var}(\hat{u}_{it}^r).$$

I restrict the sample to firms with at least five valid observations and winsorize the estimated variances at the 1st and 99th percentiles.

To examine the relationship between markups and idiosyncratic risk, I estimate the following regressions

$$\begin{aligned} \log(\mu_i) &= a_c + d_{s(i)}^c + b_c z_{c,i} + e_i^c, \\ \log(\mu_i) &= a_r + d_{s(i)}^r + b_r z_{r,i} + e_i^r, \end{aligned}$$

where μ_i denotes firm i 's winsorized average markup, and $d_{s(i)}$ denotes sector fixed effects. The variables $z_{c,i}$ and $z_{r,i}$ are standardized log cost and revenue volatility, respectively. Each is constructed by taking the logarithm of the corresponding firm-level variance, subtracting its cross-firm mean, and dividing by its cross-firm standard deviation. The coefficients b_c and b_r measure the relationship between log markups and cost and revenue volatility, respectively, per standard deviation.

Markups and Risk		
	(1)	(2)
	Cost Volatility	Revenue Volatility
Risk Proxy	0.0297*** (0.0040)	-0.0040 (0.0040)
Sector fixed effects	Yes	Yes
Observations	10,619	10,619
R^2	0.1103	0.1057

Table 1: The dependent variable is log markup. The reported coefficient is the coefficient on the standardized log cost volatility in column (1) and standardized log revenue volatility in column (2). Robust standard errors are reported in parentheses. *** denotes significance at the 1% level.

Table 1 reports the estimates. In column (1), a one-standard-deviation increase in log cost volatility is associated with a 3 percent higher markup. In column (2), the coefficient on revenue volatility is small, negative, and statistically insignificant. Interpreting cost and

revenue volatility as proxies for supply- and demand-side uncertainty, respectively, these results suggest that markups are positively related to supply risk but show no significant relationship with demand risk. These results are descriptive and serve as motivating evidence rather than establishing a causal relationship.

3 Model

Time t is discrete. There is a representative consumer with quadratic preference in firms' output and linear disutility in labor. The economy consists of N firms, indexed by $i \in \{1, \dots, N\}$, each producing a differentiated variety in quantity q_{it} and setting a price p_{it} at time t . I denote the vector of output at time t by $\mathbf{q}_t = (q_{1t}, \dots, q_{Nt})^\top \in \mathbb{R}_+^N$ and the price vector by $\mathbf{p}_t = (p_{1t}, \dots, p_{Nt})^\top \in \mathbb{R}_+^N$.

3.1 The Representative Consumer

The representative consumer has preference

$$U(\mathbf{q}_t, h_t) = \mathbf{b}_t^\top \mathbf{q}_t - \frac{1}{2} \mathbf{q}_t^\top \Gamma^{-1} \mathbf{q}_t - h_t,$$

where $\mathbf{b}_t \in \mathbb{R}^N$ is a vector of stochastic demand intercepts, h_t is the labor supply, and Γ is an $N \times N$ positive definite substitution matrix.

The consumer maximizes utility subject to the budget constraint

$$\mathbf{p}_t^\top \mathbf{q}_t = w_t h_t + \Pi_t,$$

where w_t is the wage rate, and Π_t denotes the rebated profits from all firms. I choose labour as the numeraire and I normalize the wage $w_t = 1$.

Timing I assume that quantity and labor decisions are made after the realization of shocks. Formally, the consumer observes the price vector $\mathbf{p}_t$ and the realized demand shifter $\mathbf{b}_t$ before choosing $\mathbf{q}_t$ and h_t to maximize the utility.

Optimal Choices and Demand System Substituting the budget constraint into the utility function, the first-order conditions yield the linear demand system

$$\mathbf{q}_t = \Gamma (\mathbf{b}_t - \mathbf{p}_t).$$

The derivation can be found in the Appendix B. Firm i 's demand at time t , q_{it} , depends on own- and cross-prices, as well as on a firm-specific demand intercept $d_i \equiv \Gamma_i \mathbf{b}_t$, where Γ_i denotes the i -th row of the substitution matrix Γ . Price sensitivities are similarly governed by Γ . Specifically, the off-diagonal elements characterize the network relationships between firms. For $i \neq j$, a negative (positive) Γ_{ij} implies that firms i and j are network substitutes (complements).

3.2 Firms

There are N firms. Each firm is managed by a manager. The production function of firm i is

$$q_{it} = \frac{h_{it}}{c_{it}},$$

where q_{it} is firm i 's output, h_{it} is the labour hired by firm i and c_{it} is the stochastic real marginal cost of firm i .

The firm faces a linear demand system which derives from the representative consumer problem

$$q_{it} = \Gamma_i (\mathbf{b}_t - \mathbf{p}_t).$$

I assume the demand intercepts, $\mathbf{b}_t$, and marginal costs, $\mathbf{c}_t$, are independently distributed with diagonal covariance matrices Σ_b and Σ_c , respectively. Their expected values are governed by a common factor structure, such that $\bar{\mathbf{b}}_t = \Phi_b \mathbf{f}_t$ and $\bar{\mathbf{c}}_t = \Phi_c \mathbf{f}_t$. Here, $\Phi_b, \Phi_c \in \mathbb{R}^{N \times F}$ are matrices of firm-specific factor loadings, and the F -dimensional factor $\mathbf{f}_t$ follows a stationary process.

Information As my primary focus is on how idiosyncratic shocks affect prices, I assume that managers observe the realization of the aggregate factors $\mathbf{f}_t$ prior to making their intraperiod pricing decisions. Consequently, the conditional means $\bar{\mathbf{b}}_t$ and $\bar{\mathbf{c}}_t$ perfectly absorb all aggregate fluctuations, meaning the residual uncertainties parameterized by Σ_b and Σ_c are strictly idiosyncratic.

While the model environment is dynamic, the absence of endogenous state variables implies that managers solve a sequence of intraperiod optimization problems. All aggregate dynamics stem solely from the stationary exogenous process, $\mathbf{f}_t$.

Firm Objective I assume that firms engage in Bertrand (price) competition and that managers maximize a mean-variance objective over real profits

$$\pi_{it} = (p_{it} - c_{it}) q_{it}.$$

Rather than a single risk penalty, I allow for source-dependent preferences. Specifically, I allow managers to have distinct penalties, λ_{ci} and λ_{di} , to profit volatility arising from cost versus demand shocks. I provide a rigorous microfoundation for these source-dependent penalties in a later section by modeling the underlying principal-agent problem.

To operationalize this source dependency, I must isolate the firm's exposure to each source of risk. Because realized profit is the product of a stochastic markup and a stochastic quantity, its exact variance nonlinearly compounds the underlying shocks. To maintain tractability and clarity, I approximate the profit function to a first order around expected marginal cost, $\bar{c}_{it}$, and expected demand, $\bar{q}_{it} = \Gamma_i(\bar{\mathbf{b}}_t - \mathbf{p}_t)$. Defining the mean-zero shocks $\tilde{c}_{it} \equiv c_{it} - \bar{c}_{it}$ and $\tilde{q}_{it} \equiv q_{it} - \bar{q}_{it}$, the linearized profit is⁴

$$\pi_{it} \approx \mathbb{E}_t[\pi_{it}] - \bar{q}_{it}\tilde{c}_{it} + (p_{it} - \bar{c}_{it})\tilde{q}_{it}.$$

Defining the idiosyncratic demand intercept shocks as $\tilde{\mathbf{b}}_t \equiv \mathbf{b}_t - \bar{\mathbf{b}}_t$, it follows directly from the linear demand system that $\tilde{q}_{it} = \Gamma_i\tilde{\mathbf{b}}_t$. This implies that the idiosyncratic quantity shocks are purely exogenous and are entirely independent of the prices set by any firm in the market.

Given the independence of cost and demand shocks, the variance of this linearized profit naturally separates into two components. I define the profit variance attributable to cost (supply) and demand shocks, respectively, as

$$\text{Var}_t^{(c)}(\pi_{it}) = \bar{q}_{it}^2 \sigma_{ci}^2, \quad \text{Var}_t^{(d)}(\pi_{it}) = (p_{it} - \bar{c}_{it})^2 \sigma_{qi}^2,$$

where $\sigma_{qi}^2 = \Gamma_i \Sigma_b \Gamma_i^\top$ is the variance of output driven by demand shocks.

Taking competitors' prices $\mathbf{p}_{-it}$ as given, firm i 's maximization problem is

$$\begin{aligned} \max_{p_{it}} \quad & \mathbb{E}_t[\pi_{it}] - \frac{\lambda_{ci}}{2} \text{Var}_t^{(c)}(\pi_{it}) - \frac{\lambda_{di}}{2} \text{Var}_t^{(d)}(\pi_{it}), \\ \text{s.t.} \quad & \bar{q}_{it} = \Gamma_i(\bar{\mathbf{b}}_t - \mathbf{p}_t), \\ & \text{given } \mathbf{p}_{-it}. \end{aligned}$$

The firm's optimization problem reveals a fundamental tension in managing source-

⁴Here, I disregard the term $\tilde{c}_{it}\tilde{q}_{it}$ as firms have no control over these random variables.

dependent risks. Raising the price lowers the expected quantity, $\bar{q}_{it}$, which shields the firm from supply-side volatility because exposure to cost shocks scales with the squared quantity. Conversely, a higher price increases the firm's expected margin, $(p_{it} - \bar{c}_{it})$, which exacerbates its vulnerability to demand shocks, as the demand-driven profit variance scales with the squared margin. Consequently, the optimal pricing decision systematically departs from the standard Bertrand benchmark.

Optimal Prices and Quantities I now solve for the optimal prices. Let $\bar{m}_{it} \equiv p_{it} - \bar{c}_{it}$ denote the expected margin. Differentiating the firm's objective function with respect to the real price p_{it} yields the first-order condition

$$\bar{m}_{it}^* = \frac{\theta_i + \lambda_{ci}\sigma_{ci}^2}{1 + \lambda_{di}\theta_i\sigma_{qi}^2} \bar{q}_{it}^*,$$

where $\theta_i \equiv \frac{1}{\Gamma_{ii}}$ is the inverse of the own-price demand slope. The derivation is included in the Appendix B. To characterize the firm's pricing tractably, I take a first-order approximation of the optimal margin around the risk-neutral benchmark ($\lambda_{ci} = \lambda_{di} = 0$)

$$\bar{m}_{it}^* \approx (\theta_i + \lambda_{ci}\sigma_{ci}^2 - \lambda_{di}\theta_i^2\sigma_{qi}^2) \bar{q}_{it}^*.$$

I define the term in parentheses as the firm i *risk-adjusted conduct parameter*

$$\Delta_i \equiv \theta_i + \lambda_{ci}\sigma_{ci}^2 - \lambda_{di}\theta_i^2\sigma_{qi}^2.$$

Consequently, the optimal expected margin can be expressed as

$$\bar{m}_{it}^* \approx \Delta_i \bar{q}_{it}^*.$$

This risk-adjusted conduct parameter, Δ_i captures the firm's three pricing components:

1. Risk-free Bertrand (θ_i): In the absence of risk, Δ_i collapses to the standard inverse demand slope.
2. Cost-risk Wedge ($\lambda_{ci}\sigma_{ci}^2$): Because exposure to cost volatility scales with the total quantity produced, a manager averse to cost risk seeks to restrict output. They achieve this by raising the price and through the demand system reduces the expected quantity.
3. Demand-risk Wedge ($-\lambda_{di}\theta_i^2\sigma_{qi}^2$): Note that exposure to demand-side volatility scales with the expected margin itself. To reduce profits volatility from demand shocks, a

manager reduces the price to decrease the expected margin. The θ_i^2 term translates the variance in quantity into its margin-equivalent impact.

3.3 Equilibrium

In this subsection, I define the equilibrium concept before solving for equilibrium prices and quantities.

Definition 1 (General Equilibrium). *Given a realization of aggregate factors $\mathbf{f}_t$, a stochastic general equilibrium consists of a price vector $\mathbf{p}_t$, expected quantities $\bar{\mathbf{q}}_t$, realized quantities $\mathbf{q}_t$, and aggregate labor supply L_t such that:*

1. **Firm Optimization (Bertrand-Nash):** *Conditional on the expectations $(\bar{\mathbf{b}}_t, \bar{\mathbf{c}}_t)$ and the wage w_t , but prior to the realization of the idiosyncratic shocks $(\tilde{\mathbf{b}}_t, \tilde{\mathbf{c}}_t)$, each firm i 's price p_{it} and expected quantity $\bar{q}_{it}$ maximize its mean-variance objective, taking competitors' prices $\mathbf{p}_{-it}$ as given.*
2. **Consumer Optimization:** *Following the realization of the idiosyncratic shocks $\mathbf{b}_t$ and $\mathbf{c}_t$, the realized quantity vector $\mathbf{q}_t$ and labor supply L_t maximize the representative consumer's utility, subject to their budget constraint, given the price vector $\mathbf{p}_t$ and aggregate profits Π_t .*
3. **Market Clearing:**
 - *Product markets: The quantity produced by each firm $i \in \{1, \dots, N\}$ exactly equals the realized demand of the consumer.*
 - *Labor market: The aggregate labor supplied by the consumer equals the total labor demanded by all firms for production.*

With the equilibrium defined, I solve for the expected baseline quantity vector, $\bar{\mathbf{q}}_t^*$, in terms of the fundamental $\mathbf{b}_t, \mathbf{c}_t$ and structural parameters. Using the definition of the expected margin, the first-order condition implies

$$p_{it}^* = \bar{c}_{it} + \Delta_i \bar{q}_{it}^*.$$

Let $\mathbf{\Delta} \equiv \text{diag}(\Delta_1, \dots, \Delta_N)$ denote the $N \times N$ diagonal matrix of risk-adjusted wedges. Vectorizing the pricing rule across all N firms yields

$$\mathbf{p}_t^* = \bar{\mathbf{c}}_t + \mathbf{\Delta} \bar{\mathbf{q}}_t^*.$$

I substitute this aggregate pricing behavior into the expected demand system, $\bar{\mathbf{q}}_t = \Gamma(\bar{\mathbf{b}}_t - \mathbf{p}_t)$. Rearranging the system to isolate $\bar{\mathbf{q}}_t^*$ yields the closed-form solution for the equilibrium quantities

$$\bar{\mathbf{q}}_t^* = (\Gamma^{-1} + \mathbf{\Delta})^{-1}(\bar{\mathbf{b}}_t - \bar{\mathbf{c}}_t).$$

Substituting these equilibrium quantities back into the vectorized pricing rule yields the equilibrium real prices

$$\mathbf{p}_t^* = \bar{\mathbf{c}}_t + \mathbf{\Delta}(\Gamma^{-1} + \mathbf{\Delta})^{-1}(\bar{\mathbf{b}}_t - \bar{\mathbf{c}}_t).$$

The equilibrium quantity vector maps the net fundamentals, $\bar{\mathbf{b}}_t - \bar{\mathbf{c}}_t$, through the matrix $(\Gamma^{-1} + \mathbf{\Delta})^{-1}$. This matrix governs how fundamentals propagate through the market, a process shaped by two distinct forces. The first is standard imperfect competition, determined by how easily consumers substitute between products (captured by Γ^{-1}) and the baseline markups firms charge in the absence of risk (the θ_i term within $\mathbf{\Delta}$). The second force, which is the focus of this paper, reflects risk management. It is shaped by how individual firms distort their markups to mitigate exposure to cost and demand uncertainties.

Intuition via First-order Approximation To build intuition for how risk distorts equilibrium quantities, I linearize the equilibrium quantity vector around the risk-neutral benchmark ($\lambda_{ci} = \lambda_{di} = 0$). In the absence of risk penalties, the risk-adjusted conduct parameters collapses to the standard inverse demand slope, θ_i . Let $\Theta \equiv \text{diag}(\boldsymbol{\theta})$. The risk-free equilibrium quantity vector is

$$\bar{\mathbf{q}}_t^0 = (\Gamma^{-1} + \Theta)^{-1}(\bar{\mathbf{b}}_t - \bar{\mathbf{c}}_t).$$

To understand how the risk penalties affect the equilibrium, I define a risk perturbation to the conduct parameter

$$d\Delta_i \equiv \lambda_{ci}\sigma_{ci}^2 - \lambda_{di}\theta_i^2\sigma_{qi}^2.$$

Let $d\mathbf{\Delta} \equiv \text{diag}(d\Delta_1, \dots, d\Delta_N)$. Applying a first-order Taylor expansion to the matrix inverse, $(\Gamma^{-1} + \Theta + d\mathbf{\Delta})^{-1} \approx (\Gamma^{-1} + \Theta)^{-1} - (\Gamma^{-1} + \Theta)^{-1}d\mathbf{\Delta}(\Gamma^{-1} + \Theta)^{-1}$, I can approximate the risk-adjusted equilibrium quantities as a linear deviation from the risk-free equilibrium

$$\bar{\mathbf{q}}_t^* \approx \bar{\mathbf{q}}_t^0 - (\Gamma^{-1} + \Theta)^{-1}d\mathbf{\Delta}\bar{\mathbf{q}}_t^0.$$

Defining $\Psi \equiv (\Gamma^{-1} + \Theta)^{-1}$ as the risk-free pass-through matrix, I can express the first-order approximation for firm i 's output as

$$\bar{q}_{it}^* \approx \bar{q}_{it}^0 - \sum_{j=1}^N \Psi_{ij} \bar{q}_{jt}^0 (\lambda_{cj} \sigma_{cj}^2 - \lambda_{dj} \theta_j^2 \sigma_{qj}^2).$$

This expression shows how idiosyncratic risk distorts equilibrium output through two channels. First, firm i 's output is shaped by its own risk penalties, captured by the positive direct effect ($\Psi_{ii} > 0$). Heightened cost risk induces the firm to charge a higher price, which compresses its expected quantity, whereas demand risk operates in the opposite direction. Second, firm i 's output is simultaneously affected by the risk-adjusted prices of other firms, captured by Ψ_{ij} . Crucially, the direction of these spillovers is dictated by the underlying product network. Because the pass-through term Ψ_{ij} generally inherits the sign of the underlying substitution parameter Γ_{ij} , the spillover effect hinges on whether the varieties are network substitutes or complements. If goods i and j are substitutes ($\Psi_{ij} < 0$), an increase in firm j 's cost-risk induces it to raise its price and restrict output. This shifts consumer demand toward firm i and increases $\bar{q}_{it}^*$. Conversely, if the goods are complements ($\Psi_{ij} > 0$), firm j 's risk-driven contraction directly depresses the demand for firm i 's product.

4 Markups and Risk Decomposition

This section analyzes the impact of idiosyncratic risk on equilibrium markups. To unpack the transmission channels of uncertainty, I decompose the firm's equilibrium markup into three additive components. The first is the risk-neutral markup, a counterfactual equilibrium markup in a risk-neutral world. The second is a direct *risk premium*, reflecting the firm's compensation for bearing its own cost and demand volatility. The third is a *risk-market power* component, which captures the strategic part that arise when competitors alter their output in response to risk.

The motivation for this decomposition lies in its implications for competition policy. The risk-market power component is fundamentally a byproduct of oligopolistic interactions. As I establish below, it vanishes entirely in the limit as market power dissipates ($\Gamma_{ii} \rightarrow \infty$). Consequently, if this strategic component dominates the overall risk-induced markup, resolving the underlying oligopolistic distortion concurrently mitigates the inefficiencies generated by idiosyncratic risk. In such environments, conventional antitrust enforcement is sufficient to address both sources of inefficiency, obviating the need for risk-specific policy instruments.

I define the realized markup of firm i , μ_{it}^* , as the ratio of its equilibrium price to its

realized marginal cost

$$\mu_{it}^* = \frac{p_{it}^*}{c_{it}}.$$

Letting $\boldsymbol{\mu}_t^*$ denote the corresponding vector of markups, it is important to note that $\boldsymbol{\mu}_t^*$ is a random vector. Because firms commit to prices before idiosyncratic uncertainty is resolved, the realized marginal costs c_{it} are stochastic, making the ex-post markups stochastic as well.

4.1 Markup Decomposition

I show that the vector of equilibrium markups can be decomposed into three components

$$\boldsymbol{\mu}_t^* = \underbrace{\boldsymbol{\mu}_t^0}_{\text{Risk-neutral markup}} + \underbrace{\boldsymbol{\mu}_t^{rp}}_{\text{Risk premium}} + \underbrace{\boldsymbol{\mu}_t^{rmp}}_{\text{Risk-market-power markup}}.$$

The first term, $\boldsymbol{\mu}_t^0$, represents the risk-neutral markups that would otherwise be observed in an economy where all firms are risk-neutral. The second term, $\boldsymbol{\mu}_t^{rp}$, is the pure risk premium, compensating firms for bearing both cost and demand idiosyncratic shocks. The final term, $\boldsymbol{\mu}_t^{rmp}$, captures the network effect of responses to risk on the residual demand curve faced by each firm.

To formally establish this decomposition, I proceed in two steps. First, I construct a counterfactual in which firm i is assumed to be risk-neutral, holding the expected quantity choices of rival firms fixed at their equilibrium levels. This exercise isolates the risk premium, representing the compensation required by firm i to bear uncertainty. Second, I evaluate a fully risk-neutral benchmark wherein risk penalties are uniformly eliminated across all firms in the market. As I demonstrate below, this step captures the risk market power.

4.2 Risk Premium

I first consider a counterfactual in which firm i is assumed to be risk-neutral ($\lambda_{ci} = 0$ and $\lambda_{di} = 0$), holding the prices of other firms fixed at their equilibrium levels $\mathbf{p}_{-it}^*$. From the demand system, firm i 's expected residual demand can be inverted as

$$p_{it} = d_{it}^* - \theta_i \bar{q}_{it},$$

where $d_{it}^* = \bar{b}_{it} + \sum_{j \neq i} \frac{\Gamma_{ij}}{\Gamma_{ii}} (\bar{b}_{jt} - p_{jt}^*)$ is the residual demand intercept.

The risk-neutral firm i maximizes expected profit $\bar{\pi}_{it} = (p_{it} - \bar{c}_{it})\bar{q}_{it}$. The first-order

condition yields the risk-neutral best response quantity and price

$$\bar{q}_{it}^b = \frac{d_{it}^* - \bar{c}_{it}}{2\theta_i}, \quad p_{it}^b = \bar{c}_{it} + \theta_i \bar{q}_{it}^b = \frac{d_{it}^* + \bar{c}_{it}}{2},$$

where the superscript b denotes the risk-neutral best response.

I define the *risk compensation* $p_{it}^{rp} \equiv p_{it}^* - p_{it}^b$ as the difference between the actual equilibrium price and the risk-neutral best-response price.

Proposition 1. *The risk compensation for firm i is given by*

$$p_{it}^{rp} = \frac{1}{2} (\lambda_{ci} \sigma_{ci}^2 - \lambda_{di} \theta_i^2 \sigma_{qi}^2) \bar{q}_{it}^*,$$

where $\bar{q}_{it}^*$ is the equilibrium expected quantity of firm i .

Proof. See Appendix B. □

I can draw a direct parallel between this result and the concept of the “price of risk” in the finance literature. In my framework, the effective price of risk passed to the market is scaled by a factor of 1/2. This discrepancy arises fundamentally from the presence of market power. Under linear demand, the marginal revenue curve is twice as steep as the inverse demand curve. Consequently, the firm passes on only half of the endogenous marginal cost of risk to consumers, whereas a perfectly competitive price-taking agent would demand compensation for the full marginal cost of risk.

Sign of Risk Compensation The sign of this risk compensation reflects the firm’s optimal strategy for mitigating exposure to distinct sources of uncertainty. When facing elevated cost volatility, the firm raises its price to contract quantity demanded, thereby limiting the exposure of its profits to marginal cost shocks. Consequently, cost volatility commands a positive risk compensation. Conversely, heightened demand volatility induces a negative risk compensation. Under demand uncertainty, the firm optimally lowers its price to compress the margin, which hedges profitability against fluctuations in consumer demand.

Cost and Demand Risk Premium I define the *risk premium* markup component as

$$\mu_{it}^{rp} = \frac{p_{it}^{rp}}{c_{it}}.$$

This term isolates the excess margin required to compensate the firm for source-dependent volatility.

To disentangle the risk premium arising from distinct sources of uncertainty, I formally decompose it into a cost risk premium and a demand risk premium, defined respectively as

$$\begin{aligned}\mu_{it}^{rp,c} &= \frac{1}{2} \lambda_{ci} \sigma_{ci}^2 \frac{q_{it}^*}{c_{it}}, \\ \mu_{it}^{rp,d} &= \frac{1}{2} \lambda_{di} \theta_i^2 \sigma_{qi}^2 \frac{q_{it}^*}{c_{it}}.\end{aligned}$$

4.3 Risk-Market-Power Markup

To capture the general equilibrium effects of risk on pricing behavior, I must account for how rivals' risk penalties alter the competitive environment. Let $\mathbf{p}_t^0$ denote the equilibrium price vector when all firms are strictly risk-neutral, given by

$$\mathbf{p}_t^0 = \bar{\mathbf{c}}_t + \Theta \bar{\mathbf{q}}_t^0, \quad \text{where} \quad \bar{\mathbf{q}}_t^0 = (\Gamma^{-1} + \Theta)^{-1}(\bar{\mathbf{b}}_t - \bar{\mathbf{c}}_t).$$

I define the deviation of the risk-neutral best-response price from this benchmark as the *risk strategic price wedge*

$$\mathbf{p}_t^{rmp} \equiv \mathbf{p}_t^b - \mathbf{p}_t^0.$$

This wedge isolates the pricing effects arising directly from product network competition. Let $\mathbf{d}_t^*$ and $\mathbf{d}_t^0$ denote the vectors of residual demand intercepts evaluated at the risk-adjusted equilibrium and the fully risk-neutral equilibrium, respectively. Because p_{it}^b and p_{it}^0 are best-response prices to $\mathbf{p}_{-it}^*$ and $\mathbf{p}_{-it}^0$, they can be expressed as functions of their residual demands

$$\mathbf{p}_t^b = \frac{\bar{\mathbf{c}}_t + \mathbf{d}_t^*}{2}, \quad \mathbf{p}_t^0 = \frac{\bar{\mathbf{c}}_t + \mathbf{d}_t^0}{2}.$$

Consequently, the risk strategic price wedge simplifies to the difference in these intercepts, formalized in the following proposition.

Proposition 2. *The risk strategic price wedge of firm i is*

$$p_{it}^{rmp} = \frac{d_{it}^* - d_{it}^0}{2} = \frac{1}{2} \sum_{j \neq i} \frac{\Gamma_{ij}}{\Gamma_{ii}} (p_{jt}^0 - p_{jt}^*).$$

Proof. See Appendix B. □

This result demonstrates that the network effect of risk on prices operates via shifts in residual demand as all firms simultaneously adjust their prices in response to volatility.

Sign of Risk Strategic Price Wedge The sign of the risk strategic price wedge depends on the firm's position in the product network and the pricing responses of its rivals. If network substitutes charge prices above their risk-neutral levels ($p_{jt}^* > p_{jt}^0$), the residual demand of the focal firm expands. This outward shift in residual demand increase the effective market power, inducing the focal firm to raise its own price. Thus, a firm can capture higher markups by free-riding on the risk-induced output contractions of its competitors.

I now investigate two extreme cases, perfect competition and monopoly.

Perfect Competition The following lemma establishes that this wedge vanishes under perfect competition.

Lemma 1. *The risk strategic price wedge vanishes under perfect competition*

$$\lim_{\Gamma_{ii} \rightarrow \infty} p_{it}^{rmp} = \lim_{\Gamma_{ii} \rightarrow \infty} \frac{1}{2} \sum_{j \neq i} \frac{\Gamma_{ij}}{\Gamma_{ii}} (p_{jt}^0 - p_{jt}^*) = 0.$$

Proof. See Appendix B. □

Under perfect competition, firms act strictly as price takers. Absent market power, any deviation from the risk-neutral price reflects direct compensation for bearing risk rather than strategic spillovers.

Monopoly Conversely, if firm i operates as a pure monopoly, the cross-price sensitivities Γ_{ij} equal zero for all $j \neq i$. It follows immediately that the risk strategic price wedge evaluates to zero. Intuitively, a monopolist faces no product network competition. Because residual demand is strictly independent of rival pricing, the strategic pricing mechanism is absent. This eliminates the risk strategic price wedge, meaning any adjustment from the risk-neutral baseline is driven solely by the firm's own risk compensation.

Risk-market-power markup Finally, I define the *risk-market-power markup* for firm i as

$$\mu_{it}^{rmp} = \frac{p_{it}^{rmp}}{c_{it}}.$$

Implications for Antitrust Policy The dependence of the risk-market-power markup on the cross-price sensitivities establishes that this distortion arises from oligopoly competition. If empirical evaluation reveals that this strategic component dominates the overall risk markup, the policy prescription is simple. Policymakers do not need to invent new tools

to address price distortions stemming from idiosyncratic uncertainty. Instead, standard antitrust enforcement that reduces market concentration naturally strengthen product network competition. Such policies simultaneously mitigate both the traditional market power and the strategic component of distortion due to risk.

5 Data

Estimating the structural model requires both firm level financial data and product network linkages. I utilize Compustat for the financial statements of US public firms and the Hoberg and Phillips Text based Network Industry Classification (TNIC) dataset to construct the corresponding product network.

5.1 Financial Data and Sample Construction

My empirical analysis relies on annual firm level data from Compustat covering the period 1990 to 2020. I conclude the sample in 2020 to prevent the shocks of the Covid pandemic from contaminating the structural estimates.

I restrict the sample to firms with valid identifiers (GVKEY) and at least five years of observations. Following standard practice, I exclude highly regulated utilities (SIC codes 4900 to 4999) and financial institutions (SIC codes 6000 to 6999), as their regulatory environments and accounting practices fundamentally differ from those of standard operating firms.

The primary performance measures are reported revenue (REV_{it}) and the cost of goods sold ($COGS_{it}$). I drop any firm year observations reporting anomalous negative values for either of these measures. Consistent with the abstraction of dynamic capital accumulation in my theoretical framework, I do not incorporate capital expenditure data.

I define firm level realized profit as operating revenue less the cost of goods sold

$$\pi_{it} = REV_{it} - COGS_{it}.$$

Correspondingly, I proxy the realized markup as the ratio of revenue to cost of goods sold

$$Markup_{it} = \frac{REV_{it}}{COGS_{it}}.$$

To ensure my estimation is not driven by extreme outliers or measurement error in the accounting data, I trim the bottom and top 1 percent of the $Markup_{it}$ distribution.

I present summary statistics for my sample of 16,077 firms between 1990 and 2020. While my naive markup measure yields higher values than recent estimates based on the production function approach (Benkard et al., 2025), it remains comparable to the findings of De Loecker et al. (2020). The revenue-weighted average markup in my sample is 67 percent.

Summary Statistics

Variable	Mean	Size-wt Mean	Std. Dev.	10th Pct	Median	90th Pct
Profit	854.65	18978.10	4087.69	0.51	47.72	1316.18
Revenue	2648.31	73408.66	13689.29	1.52	140.62	4107.40
COGS	1793.67	54430.56	10397.70	0.59	77.44	2587.03
Markup	1.97	1.67	1.03	1.17	1.58	3.48

Table 2: This table displays the summary statistics of the financial variables and the markups. Markup is winsorized at the 1% and 99% levels. The financial variables is expression in the unit of millions.

5.2 Product Network Data

To construct the product competition network, I apply the characteristic based methodology of Pellegrino (20265). This approach leverages the Hoberg and Phillips Text based Network Industry Classification (TNIC) data to generate a cross price sensitivity matrix calibrated to empirical substitution elasticities.

The TNIC dataset is constructed by parsing the product descriptions located in the Business section of firms’ annual 10K filings. Using natural language processing, Hoberg and Phillips compute pairwise cosine similarity scores based on the unique vocabularies firms use to describe their products. These pairwise scores form an $N \times N$ matrix that continuously maps the product space. The numerical linkage between any two firms proxies the intensity of their product market competition. A higher text based similarity score indicates that the products are closer substitutes in characteristic space, meaning the firms exert stronger competitive pressure on one another. This continuous measure offers a significant advantage over traditional classification systems like SIC or NAICS, which impose rigid boundaries and group firms based on production processes rather than final product substitutability.

Following the characteristic based microfoundation developed by Pellegrino (2025), the inverse substitution matrix of the linear demand system takes the form

$$\Gamma_t^{-1} = (1 - \alpha)\mathbf{I} + \alpha CS_t,$$

where CS_t is the cosine similarity matrix from the TNIC data and $\alpha \in [0, 1]$ is a scaling parameter. Pellegrino (2025) calibrates this parameter to match well estimated micro cross

price elasticities, yielding an estimate of $\alpha = 0.12$. I adopt this identical parameterization for my baseline specification.

As my estimation strategy requires a stable product space (factor structure), I compute the time averaged inverse substitution matrix over the sample period

$$\begin{aligned}\Gamma^{-1} &\equiv \frac{1}{T} \sum_{t=1}^T \Gamma_t^{-1} \\ &= \mathbf{I} + \alpha \frac{1}{T} \sum_{t=1}^T CS_t.\end{aligned}$$

This time invariant matrix establishes the baseline product market competition throughout the empirical analysis.

To validate my empirical framework, I benchmark my estimated demand elasticities against the same set of untargeted micro estimates evaluated by Pellegrino (2025). Despite the structural assumption of a time invariant product network, the magnitudes of my estimates remain highly consistent with the micro data. I note a specific deviation regarding the cross elasticity between Kellogg and Quaker Oats. The acquisition of Quaker Oats by PepsiCo introduces an distortion into the text based substitution matrix for these entities. For full transparency, I retain and report this estimate in my baseline results.

Demand Elasticities				
Market	Firm i	Firm j	Micro estimate	Model $\frac{\partial \log q_i}{\partial \log p_j}$
<i>Own elasticities</i>				
Auto	Ford	Ford	−4.320	−5.221
Auto	General Motors	General Motors	−6.433	−4.816
Auto	Toyota	Toyota	−3.085	−4.314
Cereals	Kellogg’s	Kellogg’s	−3.231	−2.038
Cereals	Quaker Oats	Quaker Oats	−3.031	−1.919
Computers	Apple	Apple	−11.979	−2.442
Computers	Dell	Dell	−5.570	−4.191
<i>Cross elasticities</i>				
Auto	Ford	General Motors	0.034	0.068
Auto	Ford	Toyota	0.007	0.008
Auto	General Motors	Ford	0.065	0.061
Auto	General Motors	Toyota	0.008	0.003
Auto	Toyota	Ford	0.018	0.008
Auto	Toyota	General Motors	0.008	0.003
Cereals	Kellogg’s	Quaker Oats	0.033	0.005
Cereals	Quaker Oats	Kellogg’s	0.046	0.006
Computers	Apple	Dell	0.018	0.013
Computers	Dell	Apple	0.027	0.021

Table 3: This table compares empirical micro estimates taken from Pellegrino (2025) with the corresponding model-implied elasticities, evaluated at the time-averaged mean.

6 Empirical Strategy

This section details my empirical strategy. I first show that the theoretical framework implies a factor structure for both profit and revenue. Then, I introduce the assumptions required for identification. Finally, I formalize the empirical procedure used to take the model to the data.

6.1 Factor Structure

Identifying the effects of idiosyncratic shocks requires the empirical separation of aggregate and idiosyncratic variations. Establishing a factor structure for observable profits and revenues allows me to extract the aggregate components using technology from the factor model literature (Bai and Ng, 2002). I now demonstrate that the factor structure of the underlying fundamentals naturally delivers this empirical framework.

Recall that the underlying demand and cost primitives follow a linear factor structure

$$b_{it} = \underbrace{\Phi_{bi}^\top \mathbf{f}_t}_{\bar{b}_{it}} + \varepsilon_{bit}, \quad c_{it} = \underbrace{\Phi_{ci}^\top \mathbf{f}_t}_{\bar{c}_{it}} + \varepsilon_{cit}.$$

Optimal pricing implies that a firm expected margin is a scalar multiple of its equilibrium quantity, and equilibrium quantities are linear in $\mathbf{f}_t$. It follows that both profits and revenues are quadratic in the factors. As I detail in the Appendix, vectorizing the squares and cross products of these factors allows me to express equilibrium profits and revenues as a linear factor model

$$\begin{aligned} \pi_t &= \underbrace{\Phi_\pi \text{vec}(\mathbf{f}_t \mathbf{f}_t^\top)}_{\bar{\pi}_t} + \boldsymbol{\eta}_t^\pi, \\ \mathbf{p}\mathbf{q}_t &= \underbrace{\Phi_{pq} \text{vec}(\mathbf{f}_t \mathbf{f}_t^\top)}_{\bar{\mathbf{p}}\mathbf{q}_t} + \boldsymbol{\eta}_t^{pq}, \end{aligned}$$

where Φ_π and Φ_{pq} are the profit and revenue factor loadings. The residuals $\boldsymbol{\eta}_t^\pi$ and $\boldsymbol{\eta}_t^{pq}$ absorb the idiosyncratic variations.

6.2 Semiparametric Assumptions

The baseline model is underidentified when relying solely on balance sheet and product network data without observed quantities. I achieve identification by assuming that factor loadings and risk penalties vary at the group level rather than the firm level.

Let the function $\mathcal{G}(i) \in \{1, \dots, G\}$ map each firm i to its respective group, with $G < N$. I impose the following restrictions

$$\begin{aligned} \Phi_{bi} &= \Phi_{b\mathcal{G}(i)}, & \Phi_{ci} &= \Phi_{c\mathcal{G}(i)}, \\ \lambda_{ci} &= \lambda_{c\mathcal{G}(i)}, & \lambda_{di} &= \lambda_{d\mathcal{G}(i)}. \end{aligned}$$

The economic intuition for this assumption centers on isolating price variation. If I were to strictly assume that all firms within a group produce identical quantities, any dispersion in within group profits would be driven entirely by margin differences. Knowing the baseline market power would then allow me to perfectly extract the risk markup components. My framework generalizes this logic. I impose limited differences in fundamentals by restricting them to vary only at the group level, yet I preserve the full richness of the product network linkages and risk heterogeneity. Hence, I allow firms in the same group to produce different quantities, but this variation is disciplined by their distinct network positions and

idiosyncratic risks.

Empirically, I partition the sample into bins defined by the interaction of broad SIC sector, growth rate, and average revenue. This binning controls for multiple dimensions of unobserved heterogeneity in factor loadings. First, conditioning on sector allows for industry specific aggregate factors. Second, binning by growth rate accommodates the tendency of rapidly expanding firms to load on distinct trend factors, such as shifting consumer demand. Finally, I account for size-dependent heterogeneity in factor exposures, allowing both the composition of relevant aggregate factors and the magnitude of the firm's factor sensitivities to vary across the size distribution. In the Appendix, I provide some evidence showing that this partition absorbs the vast majority of the cross-sectional variation in the data.

6.2.1 Example: Monopoly

In this section, I illustrate the logic using a monopoly example. If firms are monopoly, then their profits are

$$\pi_{it} = \frac{\Delta_i}{(1 + \Delta_i)^2} (\bar{b}_{it} - \bar{c}_{it})^2.$$

My assumptions say that some firms are exposed to the factors in a similar way, this means that their $\bar{b}_{it} - \bar{c}_{it}$ are the same. If I take the ratio of their profits, I get

$$\frac{\pi_{it}}{\pi_{jt}} = \frac{\Delta_i}{(1 + \Delta_i)^2} \frac{(1 + \Delta_j)^2}{\Delta_j}.$$

For illustration, I shut down the demand shocks. Expanding the Δ_i , I get

$$\frac{\pi_{it}}{\pi_{jt}} = \frac{\theta_i + \lambda_c \sigma_{ci}^2}{(1 + \theta_i + \lambda_c \sigma_{ci}^2)^2} \frac{(1 + \theta_j + \lambda_c \sigma_{cj}^2)^2}{\theta_j + \lambda_c \sigma_{cj}^2}.$$

Since I have estimates for σ_{ci}^2 and σ_{cj}^2 , I can solve for λ_c .

6.3 Identification

This section formalizes the identification of the structural parameters. I demonstrate that demand and cost risks map directly to observable data moments and the risk adjusted conduct parameter Δ . Using this mapping, I construct the empirical moment conditions required for estimation, leaving the complete formal derivations to the Appendix.

Demand Variance I identify the demand variance as

$$\sigma_{qi}^2 = \frac{D_i}{\Delta_i}, \quad \text{where} \quad D_i = \mathbb{E} \left[\left(\frac{\eta_{it}^{pq}}{p\bar{q}_{it}} \right)^2 \bar{\pi}_{it} \right].$$

The intuition for this expression relies on the mechanics of Bertrand competition. The squared ratio of idiosyncratic revenue to expected revenue isolates the relative magnitude of demand shocks. Because expected profit equals the risk adjusted wedge multiplied by the squared target quantity, scaling this relative variance by expected profit recovers the absolute variance of the demand shocks.

Cost Variance I identify the cost variance by isolating it from the aggregate profit variance

$$\sigma_{ci}^2 = \Delta_i \left(\frac{\mathbb{E}[(\eta_{it}^\pi)^2]}{\mathbb{E}[\bar{\pi}_{it}]} - D_i \right).$$

The scaled variance of idiosyncratic profit, $\mathbb{E}[(\eta_{it}^\pi)^2]/\mathbb{E}[\bar{\pi}_{it}]$ contains both demand and cost shocks. Using the independence assumption of demand and cost shocks, the total relative profit variance additively decomposes into a cost component and a demand component. By subtracting the previously identified demand component D_i from that, I remove the influence of demand shocks. The risk adjusted conduct parameter Δ_i then translates this residual profit variance back into the fundamental cost variance σ_{ci}^2 . Defining the composite data moment $K_i = \mathbb{E}[(\eta_{it}^\pi)^2]/\mathbb{E}[\bar{\pi}_{it}] - D_i$ allows me to express this relationship compactly as $\sigma_{ci}^2 = \Delta_i K_i$.

Substituting these identified variances back into Δ_i yields

$$\begin{aligned} \Delta_i &= \theta_i + \lambda_{c\mathcal{G}(i)} \sigma_{ci}^2 - \lambda_{d\mathcal{G}(i)} \theta_i^2 \sigma_{qi}^2 \\ &= \theta_i + \lambda_{c\mathcal{G}(i)} \Delta_i K_i - \lambda_{d\mathcal{G}(i)} \theta_i^2 \frac{D_i}{\Delta_i}. \end{aligned}$$

This forms a quadratic equation in Δ_i . Solving this polynomial allows me to express $\Delta_i = \Delta_i(\lambda_{c\mathcal{G}(i)}, \lambda_{d\mathcal{G}(i)})$ purely as a function of the group level risk penalties.

Finally, I utilize my identification assumption to project the expected squared target quantity onto a positive semidefinite $G \times G$ matrix X of rank F

$$\mathbb{E}[(\bar{q}_{it}^*)^2] = \text{Tr} \left[Y_i(\mathbf{\Delta}(\boldsymbol{\lambda})) X \right].$$

The matrix $Y_i(\mathbf{\Delta}(\boldsymbol{\lambda}))$ serves as a network projection mapping. It contains how firm i 's specific position within the product network, interacting with the risk markups of all competitors,

translates the group level covariance of fundamentals (captured by X) into the variance of its own expected quantity.

Because expected profit is the product of the risk adjusted wedge and the expected squared quantity, I can express the expected profit for firm i as

$$\mathbb{E}[\bar{\pi}_{it}] = \Delta_i(\lambda_{c\mathcal{G}(i)}, \lambda_{d\mathcal{G}(i)}) \text{Tr} \left[Y_i(\Delta(\lambda)) X \right].$$

Matching this moment against the observed average profits across all N firms yields an overidentified system for estimating the structural parameters, provided the necessary order condition holds

$$2G + \frac{G(G+1)}{2} \leq N.$$

This inequality compares the number of free parameters to the number of available equations. The necessary condition requires that the $2G$ risk penalties, along with the $G(G+1)/2$ free parameters of the covariance matrix Ω , do not exceed the N observed moment conditions.

6.4 Estimation

Exact estimation of the highly nonlinear structural system requires repeated inversion of an $N \times N$ matrix, which is computationally prohibitive for large networks. To restore tractability, I linearize the system around the risk free benchmark where $\lambda = 0$. Let $\mathbf{V}_\theta = (\Gamma^{-1} + \Theta)^{-1}$ denote the risk free pass through matrix, and let $\mathbf{Z}$ be the assignment matrix mapping firms to their respective groups. Defining $\bar{\mathbf{v}}_i^\top$ as the i th row of the product $\mathbf{V}_\theta \mathbf{Z}$, I construct the outer product $\bar{\mathbf{Y}}_i = \bar{\mathbf{v}}_i \bar{\mathbf{v}}_i^\top$. Let X_θ represent the best fit for the covariance matrix under the risk free restriction. Applying a first order approximation yields a linear expression for expected profit

$$\begin{aligned} \mathbb{E}[\bar{\pi}_{it}] \approx & \underbrace{\theta_i \text{Tr}(\bar{\mathbf{Y}}_i X)}_{\text{Oligopoly}} + \underbrace{(\lambda_{c\mathcal{G}(i)} \theta_i K_i - \lambda_{d\mathcal{G}(i)} \theta_i D_i) \text{Tr}(\bar{\mathbf{Y}}_i X_\theta)}_{\text{Direct}} \\ & - \underbrace{2\theta_i \sum_{j=1}^N (\lambda_{c\mathcal{G}(j)} \theta_j K_j - \lambda_{d\mathcal{G}(j)} \theta_j D_j) [\mathbf{V}_\theta]_{ij} (\bar{\mathbf{v}}_i^\top X_\theta \bar{\mathbf{v}}_j)}_{\text{Product Network Spillover}}. \end{aligned}$$

This approximation decomposes equilibrium profits into three distinct economic channels. The first term captures the baseline market power of firm i , which arises strictly from the oligopoly competition within the product network absent any risk. The second term isolates

the direct effect of idiosyncratic risk. Exposure to volatility directly distorts the perceived market power of the firm, inducing an adjustment to its own markups. The third term represents the network spillovers. Because idiosyncratic risks alter the target quantities of all competing firms, these strategic adjustments propagate through the product network and shift the effective residual demand faced by firm i .

I estimate the parameters $\{X, \lambda_{cG}, \lambda_{dG}\}$ by minimizing a least squares objective. To make the optimization problem tractable, I employ a semidefinite relaxation (SDR) approach, which convexifies the problem by dropping the rank constraint on X . Importantly, I verify ex post that the estimated matrix satisfies this rank constraint, ensuring that the relaxation is exact.

7 Welfare and Deadweight Loss

Thus far, I have remained agnostic about the origins of the risk penalties λ_{ci} and λ_{di} . This formulation is deliberate as my estimation strategy identifies these parameters directly from the data, bypassing the need for a specific microfoundation. Evaluating the welfare implications of idiosyncratic risk, however, requires taking a stand on the underlying frictions that generate these penalties.

I view these risk penalties as generated by agency frictions (Holmström and Milgrom, 1991). I formalize this in detail in the Appendix and this section aims to provide a summary. Under this framework, the firm (agent) problem maps exactly into the reduced-form specification analyzed in the model section. I solve the model in closed form, characterizing the risk penalties in terms of primitives that reflect the principal’s risk preferences and the agency costs of hidden effort. I derive the first-best allocation and provide a decomposition of the deadweight loss into distinct components driven by market power and risks.

7.1 The Principal-Agent Model

In this subsection, I outline the key actors and frictions of the principal-agent problem. A complete description of the model and proofs are relegated to the Appendix.

Managers/Agents The risk-averse representative consumer owns all the firms and acts as the principal. Lacking the skills to operate firms, the principal delegates this task to risk-averse managers. Manager i operates firm i by setting prices and exerting unobservable, costly effort to mitigate cost and demand risks. Specifically, exerting cost-risk management effort e_{ci} and demand-risk management effort e_{di} reduces the baseline volatilities, ζ_{ci}^2 and

ς_{qi}^2 , to post-effort volatilities $\sigma_{ci}^2 = \varsigma_{ci}^2/e_{ci}^\rho$ and $\sigma_{di}^2 = \varsigma_{di}^2/e_{di}^\rho$. The parameter ρ captures the efficiency of this variance-reduction technology. The effort costs, $C_i^c(e_{ci})$ and $C_i^d(e_{di})$, exhibit a constant elasticity ξ .⁵ The manager maximizes a CARA utility function over their compensation net of these effort costs, subject to the residual demand curve. In the Appendix, I show that the manager's problem can be reformulated as the reduced-form firm problem in the Model section.

The Principal As efforts are hidden, the principal cannot contract on them directly. Instead, they offer each manager a linear compensation contract tied to the firm's expected and unexpected costs and revenues. The principal chooses the sensitivities of this contract to maximize their mean-variance utility, subject to the manager's incentive compatibility and individual rationality constraints. To isolate the welfare implications of uninsurable risk, I restrict the principal from using the contracts to correct oligopoly market power. Finally, because compensation contracts are sticky in practice, I evaluate the optimal contract at the stationary mean of the underlying shocks.

Agency Friction The core friction in this environment is moral hazard. As efforts are not observable, the principal is required to pay an incentive premium to induce risk management efforts. This drives a wedge between the true and the effective effort costs perceived by the principal. This wedge is determined by the relative elasticities of risk management efforts ξ/ρ .

Risk Penalties under Optimal Contract Solving for the optimal contract, I derive the corresponding risk penalties

$$\begin{aligned}\lambda_{ci} &= \underbrace{\frac{\xi + \rho}{\xi}}_{\text{Agency Multiplier}} \times \underbrace{\eta_p}_{\text{Principal Risk Aversion}}, \\ \lambda_{di} &= \underbrace{\frac{\xi + \rho}{\xi}}_{\text{Agency Multiplier}} \times \underbrace{\frac{\eta_p \nu_{di} - \zeta_i}{(\bar{m}_i^\theta)^2}}_{\text{Principal Risk Aversion net of Option Value}},\end{aligned}$$

where η_p is the principal's absolute risk aversion. The parameter ν_{di} captures the risk-position of firm i , governing how shocks to firm i propagate through the product network. The parameter ζ_i represents the consumer's substitution option value. Finally, $(\bar{m}_i^\theta)^2$ is the squared risk-free oligopoly profit margin of firm i .

⁵I scale C_i^c by firm size and C_i^d by the firm's profit margin to reflect the greater complexity of running a larger firm.

I now discuss the determinants of the risk penalties.

- **Cost-Risk Penalty** (λ_{ci}): The cost-risk penalty reflects the principal's risk preference η_p , adjusted by the agency friction. Because aligning the manager's incentives requires an incentive premium, the principal optimally chooses to under-incentivize risk management relative to the frictionless benchmark. The degree of this distortion is captured exactly by the agency multiplier.
- **Demand-Risk Penalty** (λ_{di}): The demand-risk penalty consists of a tradeoff. The term $\eta_p \nu_{di}$ captures the principal's aversion to demand volatility. The term ζ_i reflects an *option value* to this volatility. As the consumer chooses consumption bundle after the realization of shocks, they optimally substitute away from products with bad shocks and toward products with good shocks. A risk-neutral principal benefits from this idiosyncratic demand volatility. The demand-risk penalty balances the principal's risk aversion against this substitution-driven option value.

7.2 The First-Best

To quantify the welfare losses generated by these frictions, I compare the decentralized equilibrium to the first-best. In the first-best, a central planner assigns the risk management efforts and dictates the expected quantities.

Let Ω_c and Ω_d denote the diagonal matrices containing the second-best, agency-friction inflated effort costs of cost and demand risk, respectively. The equilibrium quantities in the decentralized economy are

$$\bar{\mathbf{q}}^* = (\Gamma^{-1} + \Theta + \Omega_c - \Theta^2 \Omega_d)^{-1}(\bar{\mathbf{b}} - \bar{\mathbf{c}}).$$

In contrast, the central planner faces the true effort costs, $\Omega_c^{FB} = \Omega_c / \Upsilon$, where the scalar $\Upsilon \equiv \left(1 + \frac{\xi}{\rho}\right)^{\frac{\rho}{\xi+\rho}}$ captures the agency friction.⁶ The first-best allocation is given by

$$\bar{\mathbf{q}}^{FB} = (\Gamma^{-1} + \Omega_c^{FB})^{-1}(\bar{\mathbf{b}} - \bar{\mathbf{c}}).$$

Comparing these allocations reveals three sources of inefficiencies. First, the planner eliminates the oligopoly market power, reflected in the disappearance of the inverse demand slope, Θ . Second, the absence of an incentive premium reduces the supply-risk effort cost to its

⁶I note that $\Upsilon \leq e^{1/e} \approx 1.44$. I do not estimate Υ and my welfare quantification is based on this upperbound.

socially optimal level. Finally, the effort cost associated with demand risk vanishes as the elimination of market power drives prices close to the marginal cost.

7.3 Welfare and Deadweight Loss Decomposition

I evaluate allocations using the aggregate surplus function, $S(\Omega_c, \Omega_d, \bar{q})$, defined as consumer utility net of the social cost of risk management (the full expression is provided in the Appendix).⁷ The total deadweight loss (DWL) in the decentralized economy is the difference in surplus between the first-best allocation and the equilibrium

$$DWL = S(\Omega_c^{FB}, \Omega_d^{FB}, \bar{q}^{FB}) - S(\Omega_c, \Omega_d, \bar{q}).$$

Because aggregate surplus is nonlinear in the underlying frictions, I employ a Shapley-value approach to decompose the total inefficiency into three distinct components, oligopolistic product-network competition, cost risk and demand risk. I express this decomposition as

$$DWL = DWL^\Theta + DWL^{\Omega_c} + DWL^{\Omega_d}.$$

Here, DWL^Θ captures the welfare loss attributable to oligopoly market power, while DWL^{Ω_c} and DWL^{Ω_d} isolate the losses driven by the cost-risk and demand-risk agency frictions, respectively. The exact formulas governing this decomposition are deferred to the Appendix.

8 Results

This section presents the empirical findings from my structural estimation. My sample encompasses 16,077 Compustat firms operating between 1990 and 2020. To achieve identification, I group firms in the following way. Within each broad SIC industry (9 groups, the definition is included in the Appendix), I rank firms along 2 dimensions, their growth rate and average revenue. I split firms into 3×3 bins within each broad SIC industry. The Appendix provides robustness checks confirming that my primary findings remain stable under alternative bin classifications.

8.1 Markup Decomposition

Table 4 presents the decomposition of net markups. The baseline risk-neutral component accounts for 95.9 percent of the net markup on a size-weighted basis. This reflects that

⁷In this framework, this is equal to the total surplus as the principal is able to extract all the surplus through the intercept of the linear contract.

firm fundamentals and product network market competition are the main determinants of net markup. Idiosyncratic risk has a minor effect on the aggregate markup. Within this category, the margins operate in opposite directions. The cost risk premium increases the revenue-weighted net markup by 4.53 percent. In contrast, the demand risk premium reduces it by 0.87 percent. The risk market power component captures how uncertainty propagates through the product network. This final element accounts for 0.44 percent of the aggregate net markup.

Markup Decomposition (% of Net Markup)						
	Mean	Size-wt Mean	Std	10th Pct	Median	90th Pct
Baseline Risk-Neutral	99.08%	95.90%	4.86%	94.99%	99.28%	101.49%
Cost Risk Premium	1.09%	4.53%	2.44%	0.00%	0.08%	3.26%
Demand Risk Premium	-1.47%	-0.87%	4.26%	-3.57%	-0.02%	0.00%
Risk Market Power	1.30%	0.44%	2.43%	0.00%	0.61%	3.21%

Table 4: This table presents the decomposition of net markups into four components: Baseline Risk-Neutral, Cost Risk Premium, Demand Risk Premium, and Risk Market Power. For each component, the table reports the unweighted mean, size-weighted mean, standard deviation, 10th, 50th, and 90th percentiles across the sample.

Figure 1 plots the distributions of the risk markup components, which exhibit substantial cross-sectional variation. I begin by examining the cost risk premium. Specifically, this component has a standard deviation of 2.44 percent and a median of 0.08 percent. This variation is systematic across the size distribution. Large firms exhibit a higher cost risk premium as reflected by the size-weighted mean above the unweighted average.⁸

The demand risk premium demonstrates comparable heterogeneity, with a standard deviation of 4.26 percent, but the size relationship reverses. Small firms bear a higher demand risk premium, and risk market power is similarly concentrated within this group. This concentration occurs because large firms hold greater market power within the product network. Operating closer to a monopoly limits their exposure to the pricing adjustments of their rivals.

⁸In the Appendix, I also decompose the risk markups by revenue bin.

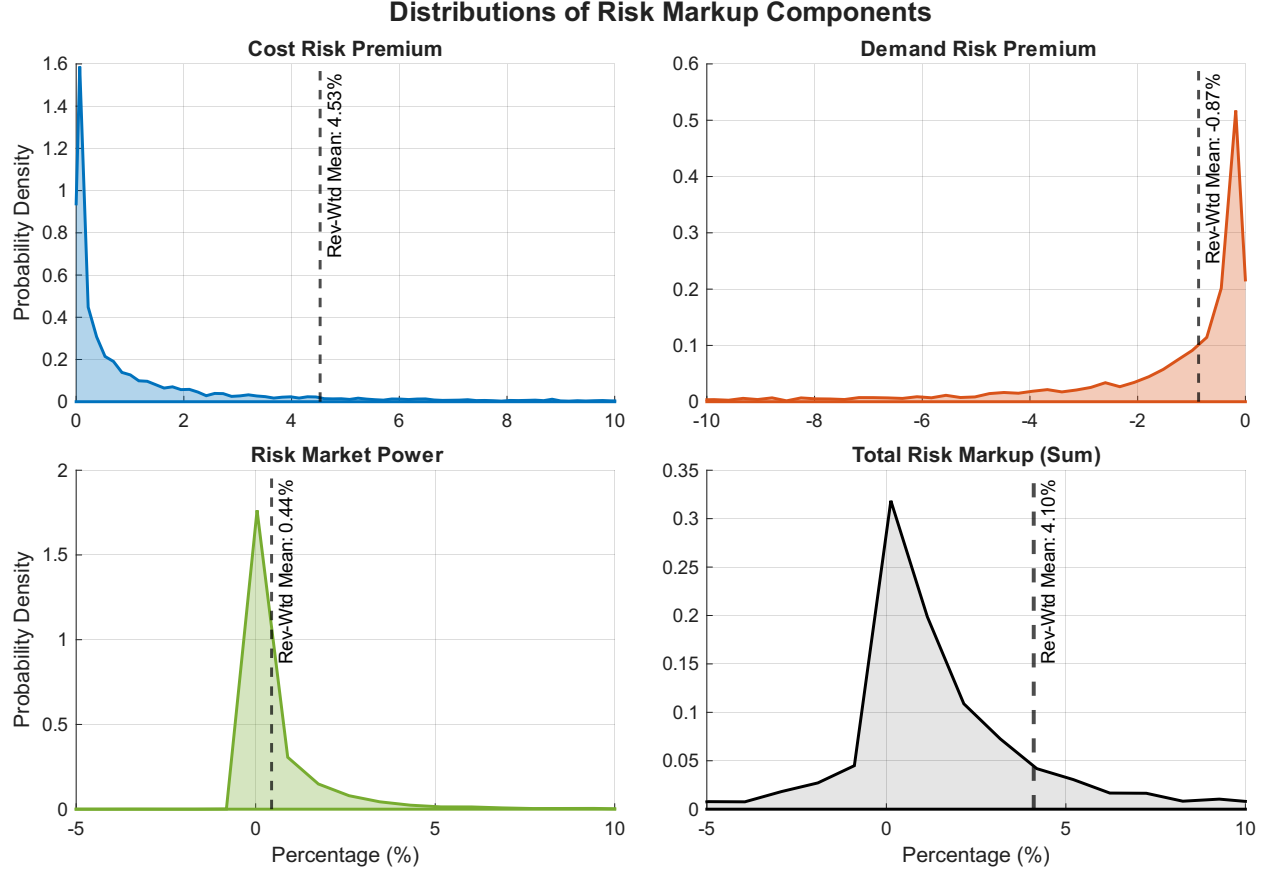


Figure 1: This figure plots the cross-sectional distributions of the cost risk premium (top left), demand risk premium (top right), risk market power (bottom left), and the total risk markup (bottom right). The dashed vertical lines denote the size-weighted mean for each component.

8.2 Welfare

Table 5 presents the welfare implications implied by my estimates. Product network competition accounts for the vast majority of the inefficiency, contributing 91.22 percent of the total DWL, or 14.67 percent of the first-best surplus. Consistent with the markup decomposition, cost risk generates a larger welfare loss than demand risk. The cost risk component comprises 8.67 percent of the total DWL (1.39 percent of the first-best surplus), whereas the demand risk component is quantitatively negligible.

Deadweight Loss Decomposition		
Component	Percentage of	
	Total DWL	First-Best Surplus
Market Power (DWL^{Θ})	91.22%	14.67%
Cost Risk (DWL^{Ω_c})	8.67%	1.39%
Demand Risk (DWL^{Ω_d})	0.11%	0.02%
Total	100.00%	16.08%

Table 5: This table decomposes the total deadweight loss (DWL) into three components: market power, cost risk, and demand risk. The first column reports each component’s share of the total DWL. The second column reports each component as a percentage of the first-best surplus.

9 Discussion

My empirical analysis reveals that idiosyncratic risk plays a modest role in explaining markups. This conclusion contrasts with Boar et al. (2026) that attributes the vast majority of observed markup to uninsurable risk. In this section, I reconcile these divergence by examining the methodological and empirical differences.

9.1 Comparison with Boar et al. (2026)

Boar et al. (2026) find that risk is the primary driver of aggregate markups among private firms. Utilizing a model of entrepreneur dynamics with financial frictions and risky production, they demonstrate that undiversifiable risk forces private entrepreneurs to operate below an efficient scale. Because of span-of-control constraints, this restricted firm size implies a higher markup. By mapping the shortfall in firm size back to its structural causes, the authors are able to decompose observed markups into their financial, span-of-control, and risk-induced components.

A central question is how to reconcile their conclusions with the quantitatively modest role for risk I document among U.S. public firms. While a natural starting point is the fundamental difference in the empirical samples, the divergence in magnitudes is equally tied to structural modeling choices.

While Boar et al. (2026) acknowledge that their findings rely on a stringent set of assumptions, I highlight an unaddressed vulnerability regarding their risk distribution. In their framework, the mapping from risk to firm size and markups is governed by the negative covariance between the entrepreneurial stochastic discount factor (SDF) and production shocks. To parsimoniously match the aggregate fat-tailed output innovation, Boar et al.

(2026) impose a single mixture-of-Gaussians distribution. However, this specific distributional choice artificially inflates the SDF covariance. This forces a large reduction in firm size and mechanically attributes a large share of the observed markup to the risk channel.

I note that a fat-tailed distribution of size-weighted aggregate output innovations is not a feature unique to private firms. I reproduce this exact pattern using the output growth rates of U.S. public firms. As Panel A of Figure 2 illustrates, a single Gaussian distribution fails to capture the fat tails of the aggregate data. However, one can match this observed aggregate distribution by allowing for cross-sectional heterogeneity rather than imposing a fat-tailed innovation shock as shown in Panel B of Figure 2. Importantly, adopting this heterogeneity reduces the magnitude of the SDF covariance, which effectively overturns the quantitative magnitude of their main result.

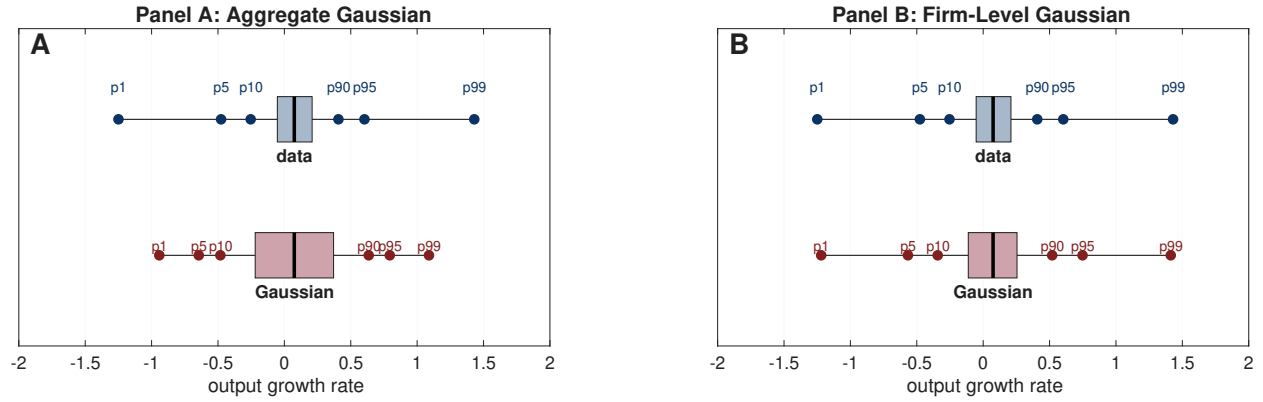


Figure 2: The top plots in both panels display the empirical, size-weighted distribution of output growth rates for U.S. public firms from 1990 to 2020. The bottom plot of Panel A plots a single, homogeneous Gaussian distribution calibrated to the empirical mean and variance of the observed data. The bottom plot of Panel B displays the aggregate distribution implied by my estimated firm-specific Gaussian innovations.

One might worry that comparing my oligopoly framework to their entrepreneur dynamic model is complicated because I abstract from financial constraints and the wealth distribution. However, Boar et al. (2026) explicitly demonstrate that financial constraints are of secondary importance and that wealth levels do not meaningfully alter the risk wedge in their setting. Unlike their framework, my empirical estimates remain entirely agnostic to these underlying microfoundations.

Finally, the fundamental difference in the underlying data samples likely contributes to the divergence in estimated magnitudes. Attempting to replicate my estimation using private firm data to verify this is not possible as my framework requires product network data which is only available for US public firms. Thus, focusing on public firms remains necessary to evaluate the joint impact of product network competition and idiosyncratic risks on markups.

10 Conclusion

In this paper, I develop a framework to study firm pricing under product network competition and uninsurable idiosyncratic risk. I propose an estimation strategy that combines panel financial statements with microdata on product market network. My estimates indicate that idiosyncratic risk plays a limited role in determining prices and generates only negligible welfare losses. The policy implication of my analysis is that regulators should maintain their focus on traditional antitrust enforcement and competition policy. As market power remains the main driver of deadweight loss, interventions aimed at mitigating uninsurable firm risk yield minimal efficiency gains.

References

- David Autor, David Dorn, Lawrence F Katz, Christina Patterson, and John Van Reenen. The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, 135(2):645–709, 05 2020. ISSN 0033-5533. doi: 10.1093/qje/qjaa004. URL <https://doi.org/10.1093/qje/qjaa004>.
- Jushan Bai and Serena Ng. Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191–221, 2002. doi: <https://doi.org/10.1111/1468-0262.00273>. URL <https://onlinelibrary.wiley.com/doi/abs/10.1111/1468-0262.00273>.
- Kristian Behrens, Giordano Mion, Yasusada Murata, and Jens Suedekum. Quantifying the gap between equilibrium and optimum under monopolistic competition. *The Quarterly Journal of Economics*, 135(4):2299–2360, 11 2020. ISSN 0033-5533. doi: 10.1093/qje/qjaa017. URL <https://doi.org/10.1093/qje/qjaa017>.
- C. Lanier Benkard, Nathan H. Miller, and Ali Yurukoglu. The rise of market power and the macroeconomic implications: Comment. Working Paper 34513, National Bureau of Economic Research, November 2025.
- Anmol Bhandari, Sanhitha Jugulum, Laura Veldkamp, and Venky Venkateswaran. Firm specialization vs. diversification. Working paper, 2026.
- Corina Boar, Denis Gorea, and Virgiliu Midrigan. The aggregate costs of uninsurable business risk. *American Economic Review*, 116(7):2604–42, July 2026. doi: 10.1257/aer.20230849. URL <https://www.aeaweb.org/articles?id=10.1257/aer.20230849>.
- Ariel T Burstein, Vasco M Carvalho, and Basile Grassi. Bottom-up markup fluctuations. *The Quarterly Journal of Economics*, 140(4):2619–2684, 11 2025. ISSN 0033-5533. doi: 10.1093/qje/qjaf029. URL <https://doi.org/10.1093/qje/qjaf029>.
- Joel M. David, Lukas Schmid, and David Zeke. Risk-adjusted capital allocation and misallocation. *Journal of Financial Economics*, 145(3):684–705, 2022. ISSN 0304-405X. doi: <https://doi.org/10.1016/j.jfineco.2022.06.001>. URL <https://www.sciencedirect.com/science/article/pii/S0304405X22001398>.
- Jan De Loecker, Jan Eeckhout, and Gabriel Unger. The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, 135(2):561–644, Jan 2020. ISSN 0033-5533. doi: 10.1093/qje/qjz041. URL <https://doi.org/10.1093/qje/qjz041>.

- Sebastian Di Tella, Cedomir Malgieri, and Christopher Tonetti. Risk markups. Working Paper, November 2025.
- Chris Edmond, Virgiliu Midrigan, and Daniel Yi Xu. How costly are markups? *Journal of Political Economy*, 131(7):1619–1675, 2023. doi: 10.1086/722986. URL <https://www.journals.uchicago.edu/doi/abs/10.1086/722986>.
- Jan Eeckhout and Laura Veldkamp. Data and markups: A macro-finance perspective. *SSRN Electronic Journal*, 2024. doi: 10.2139/ssrn.4775852.
- Dennis Epple. Hedonic prices and implicit markets: Estimating demand and supply functions for differentiated products. *Journal of Political Economy*, 95(1):59–80, 1987. doi: 10.1086/261441. URL <https://www.journals.uchicago.edu/doi/abs/10.1086/261441>.
- Gustavo Grullon, Yelena Larkin, and Roni Michaely. Are us industries becoming more concentrated? *Review of Finance*, 23(4):697–743, 07 2019. ISSN 1572-3097. doi: 10.1093/rof/rfz007. URL <https://doi.org/10.1093/rof/rfz007>.
- Thomas Hasenzagl and Luis Pérez. Precautionary pricing and markup cyclical. Working paper, August 2026.
- Gerard Hoberg and Gordon Phillips. Text-based network industries and endogenous product differentiation. *Journal of Political Economy*, 124(5):1423–1465, 2016. doi: 10.1086/688176.
- Bengt Holmström and Paul Milgrom. Multitask Principal–Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design. *The Journal of Law, Economics, and Organization*, 7:24–52, 01 1991. ISSN 8756-6222. doi: 10.1093/jleo/7.special_issue.24. URL https://doi.org/10.1093/jleo/7.special_issue.24.
- Spencer Y. Kwon, Yueran Ma, and Kaspar Zimmermann. 100 years of rising corporate concentration. *American Economic Review*, 114(7):2111–40, July 2024. doi: 10.1257/aer.20220621. URL <https://www.aeaweb.org/articles?id=10.1257/aer.20220621>.
- Kelvin J. Lancaster. A new approach to consumer theory. *Journal of Political Economy*, 74(2):132–157, 1966. doi: 10.1086/259131. URL <https://www.journals.uchicago.edu/doi/abs/10.1086/259131>.
- Bruno Pellegrino. Product differentiation and oligopoly: A network approach. *American Economic Review*, 115(4):1170–1225, April 2025. doi: 10.1257/aer.20201425. URL <https://www.aeaweb.org/articles?id=10.1257/aer.20201425>.

Sherwin Rosen. Hedonic prices and implicit markets: Product differentiation in pure competition. *Journal of Political Economy*, 82(1):34–55, 1974. doi: 10.1086/260169. URL <https://www.journals.uchicago.edu/doi/abs/10.1086/260169>.

James Traina. Is aggregate market power increasing? production trends using financial statements. Working paper, New York University (NYU) - Leonard N. Stern School of Business, February 2018. Available at SSRN.

Evgeny Zhelobodko, Sergey Kokovin, Mathieu Parenti, and Jacques-François Thisse. Monopolistic competition: Beyond the constant elasticity of substitution. *Econometrica*, 80(6):2765–2784, 2012. doi: <https://doi.org/10.3982/ECTA9986>. URL <https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA9986>.

Appendix

A Additional Figures



Figure 3: This figure plots the unweighted averages of the risk markup components. Firms are sorted into five bins based on their average revenue.

Markup Decomposition by Sector (% of Net Markup)						
Sector	Mean	Size-wt Mean	Std. Dev.	10th Pct.	Median	90th Pct.
Panel A: Baseline Risk-Neutral						
Agriculture, Forestry, Fishing	99.66	98.79	4.90	96.35	99.63	100.40
Mining	98.32	97.01	3.87	95.06	99.25	100.20
Construction	99.07	99.13	1.38	97.34	99.48	100.00
Manufacturing	98.72	95.19	4.97	94.41	99.04	100.82
Transportation, Comm, Elec, Gas	99.26	96.17	4.96	95.34	99.34	101.95
Wholesale Trade	98.22	96.73	3.21	94.67	98.94	100.09
Retail Trade	97.59	95.00	4.16	92.34	98.29	100.00
Services	100.37	98.26	5.15	96.64	99.80	104.14
Nonclassifiable	98.05	100.20	2.99	93.83	99.10	100.04
Panel B: Cost Risk Premium						
Agriculture, Forestry, Fishing	0.22	0.63	0.62	0.00	0.03	0.42
Mining	0.85	5.28	2.08	0.00	0.11	2.04
Construction	0.16	0.07	0.64	-0.00	0.00	0.23
Manufacturing	0.91	4.66	2.01	0.00	0.07	2.71
Transportation, Comm, Elec, Gas	1.35	3.78	2.90	-0.00	0.02	4.19
Wholesale Trade	1.70	6.05	3.15	0.00	0.31	5.40
Retail Trade	1.97	4.50	3.03	0.00	0.66	6.21
Services	1.10	4.67	2.70	0.00	0.02	3.37
Nonclassifiable	0.60	2.93	1.77	-0.00	0.04	1.37
Panel C: Demand Risk Premium						
Agriculture, Forestry, Fishing	-0.94	-0.01	4.36	-1.02	-0.00	0.00
Mining	-0.71	-2.81	2.47	-1.45	-0.00	-0.00
Construction	-0.18	-0.06	0.64	-0.32	-0.00	0.00
Manufacturing	-1.07	-0.22	3.81	-1.92	-0.01	0.00
Transportation, Comm, Elec, Gas	-1.74	-0.30	4.52	-4.97	-0.03	0.00
Wholesale Trade	-1.20	-3.55	3.29	-3.15	-0.08	-0.00
Retail Trade	-0.85	-0.05	3.29	-1.31	-0.00	0.00
Services	-2.53	-3.53	5.41	-8.00	-0.19	0.00
Nonclassifiable	-0.51	-3.34	2.08	-1.14	-0.00	0.00
Panel D: Risk Market Power						
Agriculture, Forestry, Fishing	1.06	0.59	1.76	0.00	0.30	2.84
Mining	1.54	0.53	3.29	0.00	0.55	3.83
Construction	0.95	0.86	1.36	0.00	0.53	2.65
Manufacturing	1.44	0.38	2.77	0.00	0.68	3.45
Transportation, Comm, Elec, Gas	1.12	0.35	2.50	0.00	0.45	2.88
Wholesale Trade	1.28	0.77	1.95	0.00	0.71	3.24
Retail Trade	1.29	0.55	1.59	0.00	0.88	3.21
Services	1.06	0.60	1.77	0.00	0.49	2.66
Nonclassifiable	1.86	0.21	2.73	0.00	0.82	5.55

Table 6: This table reports the statistics of the markup components for each sector. All values are expressed as percentages of the net markup.

B Omitted Derivation

B.1 Consumer Optimization

Substituting the budget constraint into the consumer preference yields

$$\max_{\mathbf{q}_t} \quad \mathbf{b}_t^\top \mathbf{q}_t - \frac{1}{2} \mathbf{q}_t^\top \Gamma^{-1} \mathbf{q}_t - \mathbf{p}_t^\top \mathbf{q}_t + \Pi_t.$$

The first order condition yields the demand system

$$\mathbf{q}_t = \Gamma(\mathbf{b}_t - \mathbf{p}_t).$$

B.2 Firm Optimization

Firm i problem reads

$$\begin{aligned} \max_{p_{it}} \quad & \mathbb{E}_t[\pi_{it}] - \frac{\lambda_{ci}}{2} \text{Var}_t^{(c)}(\pi_{it}) - \frac{\lambda_{di}}{2} \text{Var}_t^{(d)}(\pi_{it}), \\ \text{s.t.} \quad & \bar{q}_{it} = \Gamma_i(\bar{\mathbf{b}}_t - \mathbf{p}_t), \\ & \text{given } \mathbf{p}_{-it}. \end{aligned}$$

I first note that

$$\frac{\partial q_{it}}{\partial p_{it}} = -\Gamma_{ii}.$$

The derivatives of each component with respect to p_{it} are

$$\begin{aligned} \frac{\partial \mathbb{E}_t[\pi_{it}]}{\partial p_{it}} &= \bar{q}_{it} - (p_{it} - \bar{c}_{it})\Gamma_{ii}, \\ \frac{\partial \text{Var}_t^{(c)}(\pi_{it})}{\partial p_{it}} &= -2\Gamma_{ii}\bar{q}_{it}\sigma_{ci}^2, \\ \frac{\partial \text{Var}_t^{(d)}(\pi_{it})}{\partial p_{it}} &= 2(p_{it} - \bar{c}_{it})\sigma_{qi}^2. \end{aligned}$$

Hence, the first order condition reads

$$\bar{q}_{it}^* - (p_{it}^* - \bar{c}_{it})\Gamma_{ii} + \lambda_{ci}\Gamma_{ii}\bar{q}_{it}^*\sigma_{ci}^2 - \lambda_{di}(p_{it}^* - \bar{c}_{it})\sigma_{qi}^2 = 0.$$

Rearranging and using $m_{it}^* = (p_{it}^* - \bar{c}_{it})$, I obtain

$$m_{it}^* = \frac{\theta_i + \lambda_{ci}\sigma_{ci}^2}{1 + \lambda_{di}\theta_i\sigma_{qi}^2} \bar{q}_{it}^*$$

B.3 Proposition 1: Risk Compensation

Recall that firm i first order condition is

$$p_{it}^* = \bar{c}_{it} + (\theta_i + d\Delta_i)\bar{q}_{it}^*.$$

Next, I want to replace the residual demand intercept in the best-response price with equilibrium quantity. To achieve that, I use firm i 's first order condition and inverse residual demand $p_{it}^* = d_{it}^* - \theta_i\bar{q}_{it}^*$ to express for d_{it}^* in terms of $\bar{q}_{it}^*$

$$d_{it}^* - \bar{c}_{it} = (2\theta_i + d\Delta_i)\bar{q}_{it}^*.$$

Substituting d_{it}^* into the best-response price $p_{it}^b = \bar{c}_{it} + (d_{it}^* - \bar{c}_{it})/2$, I obtain

$$p_{it}^b = \bar{c}_{it} + \theta_i\bar{q}_{it}^* + \frac{1}{2}d\Delta_i\bar{q}_{it}^*.$$

Taking the difference between equilibrium price and best-response price yields

$$\begin{aligned} p_{it}^{rp} &= p_{it}^* - p_{it}^b \\ &= \frac{1}{2}d\Delta_i\bar{q}_{it}^*. \end{aligned}$$

B.4 Proposition 2: Risk Strategic Price Wedge

Using the definition

$$\begin{aligned} \mathbf{p}_t^{rmp} &= \mathbf{p}_t^b - \mathbf{p}_t^0 \\ &= \frac{\bar{\mathbf{c}}_t + \mathbf{d}_t^*}{2} - \frac{\bar{\mathbf{c}}_t + \mathbf{d}_t^0}{2} \\ &= \frac{\mathbf{d}_t^* - \mathbf{d}_t^0}{2}. \end{aligned}$$

B.5 Lemma 1: Risk Strategic Price Wedge under Perfect Competition

Using the result from Proposition 2,

$$\begin{aligned}\lim_{\Gamma_{ii} \rightarrow \infty} p_{it}^{rmp} &= \lim_{\Gamma_{ii} \rightarrow \infty} \frac{d_{it}^* - d_{it}^0}{2} \\ &= \lim_{\Gamma_{ii} \rightarrow \infty} \frac{1}{2} \sum_{j \neq i} \frac{\Gamma_{ij}}{\Gamma_{ii}} (p_{jt}^0 - p_{jt}^*) \\ &= 0.\end{aligned}$$

C Empirical Strategy Derivation

This appendix provides the formal derivations for the empirical strategy. I first detail how the factor structure of fundamentals translates into a factor model for profits and revenues. Second, I show how the demand and cost variances are identified directly from observable data moments. Third, I map these components through the product network to formalize the expected squared target quantities. Finally, I derive the first-order approximation used for the tractable estimation of the structural parameters.

C.1 Revenue and Profit Factor Structure

I first demonstrate that equilibrium profits and revenues inherently follow a factor structure. Recall that the expected target quantity vector is given by

$$\bar{\mathbf{q}}_t^* = (\Gamma^{-1} + \Delta)^{-1}(\bar{\mathbf{b}}_t - \bar{\mathbf{c}}_t).$$

Substituting the linear factor structure of the fundamentals yields

$$\bar{\mathbf{q}}_t^* = (\Gamma^{-1} + \Delta)^{-1}(\Phi_b - \Phi_c)\mathbf{f}_t = \tilde{H}\mathbf{f}_t,$$

where $\tilde{H} = (\Gamma^{-1} + \Delta)^{-1}(\Phi_b - \Phi_c)$ captures the firm's quantity exposure to the aggregate factors. Because optimal pricing implies that a firm's expected margin is a scalar multiple of its quantity, $\bar{\mathbf{m}}_t^* = \Delta\bar{\mathbf{q}}_t^*$, I can express the expected (or factor-driven) profit of firm i as

$$\begin{aligned}\bar{\pi}_{it} &\equiv \mathbb{E}_t[\pi_{it}] = \bar{m}_{it}^*\bar{q}_{it}^* \\ &= \Delta_i(\bar{q}_{it}^*)^2 \\ &= \Delta_i(\tilde{H}_i\mathbf{f}_t\mathbf{f}_t^\top\tilde{H}_i^\top) \\ &= \text{vec}(\Delta_i\tilde{H}_i^\top\tilde{H}_i)^\top\text{vec}(\mathbf{f}_t\mathbf{f}_t^\top),\end{aligned}$$

where $\tilde{H}_i$ is the i -th row of $\tilde{H}$. Let $\Phi_{\pi,i} = \text{vec}(\Delta_i\tilde{H}_i^\top\tilde{H}_i)^\top$ denote the profit factor loadings, and let $\mathbf{g}_t = \text{vec}(\mathbf{f}_t\mathbf{f}_t^\top)$ be the vectorized squares and cross-products of the factors. I have $\bar{\pi}_{it} = \Phi_{\pi,i}\mathbf{g}_t$. The realized profit is then

$$\pi_{it} = \bar{\pi}_{it} - \bar{q}_{it}^*\tilde{c}_{it} + \bar{m}_{it}^*\tilde{q}_{it} = \Phi_{\pi,i}\mathbf{g}_t + \eta_{it}^\pi,$$

where $\eta_{it}^\pi = -\bar{q}_{it}^*\tilde{c}_{it} + \bar{m}_{it}^*\tilde{q}_{it}$ is the mean-zero idiosyncratic profit residual.

Similarly, firm revenue is $pq_{it} = \pi_{it} + c_{it}q_{it}$. Because costs and quantities both exhibit a linear factor structure, their product (and thus overall revenue) also loads linearly onto $\mathbf{g}_t$,

allowing me to write $p_{q_{it}} = \Phi_{pq,i} \mathbf{g}_t + \eta_{it}^{pq}$.

C.2 Identification of Demand and Cost Variances

I now show how to express the demand and cost variances for firm i strictly as functions of its risk-adjusted wedge Δ_i and the observable data moments D_i and K_i .

Demand Variance Assuming prices are set ex-ante, realized revenue is $p_{q_{it}} = p_{it}(\bar{q}_{it}^* + \tilde{q}_{it})$. The ratio of the idiosyncratic revenue residual $\eta_{it}^{pq} = p_{it}\tilde{q}_{it}$ to the expected revenue $\bar{p}_{q_{it}} = p_{it}\bar{q}_{it}^*$ eliminates the sticky price: $\frac{\eta_{it}^{pq}}{\bar{p}_{q_{it}}} = \frac{\tilde{q}_{it}}{\bar{q}_{it}^*}$.

Squaring this ratio, multiplying by the expected profit $\bar{\pi}_{it} = \Delta_i(\bar{q}_{it}^*)^2$, and taking the unconditional expectation yields the observable data moment D_i , where the unobservable baseline quantity cancels out:

$$D_i \equiv \mathbb{E} \left[\left(\frac{\eta_{it}^{pq}}{\bar{p}_{q_{it}}} \right)^2 \bar{\pi}_{it} \right] = \mathbb{E} \left[\frac{\tilde{q}_{it}^2}{(\bar{q}_{it}^*)^2} \Delta_i (\bar{q}_{it}^*)^2 \right] = \Delta_i \mathbb{E}[\tilde{q}_{it}^2].$$

Since $\mathbb{E}[\tilde{q}_{it}^2] = \sigma_{qi}^2$, I identify the demand variance as

$$\sigma_{qi}^2 = \frac{D_i}{\Delta_i}.$$

Cost Variance The variance of the idiosyncratic profit residual η_{it}^π consists of both cost and demand variations:

$$\mathbb{E}[(\eta_{it}^\pi)^2] = \mathbb{E}[(\bar{q}_{it}^*)^2] \sigma_{ci}^2 + \Delta_i^2 \mathbb{E}[(\bar{q}_{it}^*)^2] \sigma_{qi}^2.$$

From the expected profit condition, I know that $\mathbb{E}[(\bar{q}_{it}^*)^2] = \frac{\mathbb{E}[\bar{\pi}_{it}]}{\Delta_i}$. Substituting this and my expression for the demand variance ($\sigma_{qi}^2 = D_i/\Delta_i$), I obtain:

$$\mathbb{E}[(\eta_{it}^\pi)^2] = \frac{\mathbb{E}[\bar{\pi}_{it}]}{\Delta_i} \sigma_{ci}^2 + \mathbb{E}[\bar{\pi}_{it}] D_i.$$

Rearranging to solve for the cost variance gives:

$$\sigma_{ci}^2 = \Delta_i \left(\frac{\mathbb{E}[(\eta_{it}^\pi)^2]}{\mathbb{E}[\bar{\pi}_{it}]} - D_i \right).$$

Defining the composite observable data moment as $K_i \equiv \frac{\mathbb{E}[(\eta_{it}^\pi)^2]}{\mathbb{E}[\bar{\pi}_{it}]} - D_i$, I compactly write $\sigma_{ci}^2 = \Delta_i K_i$.

C.3 Network Projection and Target Quantities

I now map these identified moments back into the aggregate network to express unconditional expected profits. I define $\mathbf{Z} \in \{0, 1\}^{N \times G}$ as the firm-to-group assignment matrix. I factor the fundamental loadings as $\Phi_b - \Phi_c = \mathbf{Z}\mathbf{A}$, where $\mathbf{A}$ captures group-level sensitivities.

The expected quantity vector is $\bar{\mathbf{q}}^* = (\Gamma^{-1} + \Delta)^{-1} \mathbf{Z}\mathbf{A}\mathbf{f}_t$. Defining the aggregate network matrix $\mathbf{V}(\Delta) = (\Gamma^{-1} + \Delta)^{-1}$, I let $\mathbf{v}_i^\top \in \mathbb{R}^{1 \times G}$ be the i -th row of $\mathbf{V}(\Delta)\mathbf{Z}$. Applying the cyclic property of the trace operator, I define the observable network projection matrix $\mathbf{Y}_i(\Delta) = \mathbf{v}_i\mathbf{v}_i^\top$.

Consolidating the unobservable factor covariances into a master structural matrix X , the expected squared target quantities can be expressed as:

$$\mathbb{E}[(\bar{q}_{it}^*)^2] = \text{Tr}(\mathbf{Y}_i(\Delta)X).$$

It directly follows that the unconditional expected profit for firm i is:

$$\mathbb{E}[\bar{\pi}_{it}] = \Delta_i \text{Tr}(\mathbf{Y}_i(\Delta)X). \quad (1)$$

C.4 First-Order Approximation

To estimate the system tractably, I linearize equation (1) by performing a multivariate Taylor expansion around the risk-free parameterization.

Let $\Theta = \text{diag}(\theta_1, \dots, \theta_N)$ denote the diagonal matrix of baseline wedges. Evaluated at $\Delta = \Theta$, I define:

- **Base Network Matrix:** $\mathbf{V}_\theta = (\Gamma^{-1} + \Theta)^{-1}$
- **Base Network Sensitivities:** $\bar{\mathbf{v}}_i^\top$ is the i -th row of $\mathbf{V}_\theta\mathbf{Z}$
- **Base Projection Matrix:** $\bar{\mathbf{Y}}_i = \bar{\mathbf{v}}_i\bar{\mathbf{v}}_i^\top$
- **Base Covariance:** X_θ , representing the best fit for X under the risk-free restriction.

Treating X as an independent matrix variable, the derivative of $\mathbb{E}[\bar{\pi}_{it}]$ evaluated at the base point is:

$$\left. \frac{\partial \mathbb{E}[\bar{\pi}_{it}]}{\partial X} \right|_{(\Theta, X_\theta)} = \theta_i \bar{\mathbf{Y}}_i.$$

Next, I find the derivative with respect to the j -th wedge Δ_j . Differentiating the network matrix $\mathbf{V}$ yields $\frac{\partial \mathbf{V}}{\partial \Delta_j} = -\mathbf{V}\mathbf{E}_{jj}\mathbf{V}$. By applying the chain rule to the trace term

$\text{Tr}(\mathbf{Y}_i(\mathbf{\Delta})X) = \mathbf{v}_i^\top X \mathbf{v}_i$, I obtain the partial derivatives evaluated at the base point:

$$\begin{aligned} \text{For } j = i : \quad & \left. \frac{\partial \mathbb{E}[\bar{\pi}_{it}]}{\partial \Delta_i} \right|_{(\Theta, X_\theta)} = \text{Tr}(\bar{\mathbf{Y}}_i X_\theta) - 2\theta_i [\mathbf{V}_\theta]_{ii} \bar{\mathbf{v}}_i^\top X_\theta \bar{\mathbf{v}}_i, \\ \text{For } j \neq i : \quad & \left. \frac{\partial \mathbb{E}[\bar{\pi}_{it}]}{\partial \Delta_j} \right|_{(\Theta, X_\theta)} = -2\theta_i [\mathbf{V}_\theta]_{ij} \bar{\mathbf{v}}_i^\top X_\theta \bar{\mathbf{v}}_j. \end{aligned}$$

Aggregating the first-order deviation terms from both X and $\mathbf{\Delta}$ simplifies into my linearized expected profit equation, isolating the direct wedge scaling, the structural covariance shift, and the network spillover effects:

$$\mathbb{E}[\bar{\pi}_{it}] \approx \theta_i \text{Tr}(\bar{\mathbf{Y}}_i X_\theta) + \theta_i \text{Tr}(\bar{\mathbf{Y}}_i (X - X_\theta)) + (\Delta_i - \theta_i) \text{Tr}(\bar{\mathbf{Y}}_i X_\theta) - 2\theta_i \sum_{j=1}^N (\Delta_j - \theta_j) [\mathbf{V}_\theta]_{ij} (\bar{\mathbf{v}}_i^\top X_\theta \bar{\mathbf{v}}_j).$$

Consolidating the first two terms yields $\theta_i \text{Tr}(\bar{\mathbf{Y}}_i X)$.

Finally, substituting the explicit mapping of the risk-adjusted wedge derived in the main text, $\Delta_i \approx \theta_i + \lambda_{c\mathcal{G}(i)} \theta_i K_i - \lambda_{d\mathcal{G}(i)} \theta_i^2 \frac{D_i}{\theta_i}$, into the $(\Delta_i - \theta_i)$ and $(\Delta_j - \theta_j)$ components recovers the exact linear estimation equation presented in the main text:

$$\begin{aligned} \mathbb{E}[\bar{\pi}_{it}] \approx & \underbrace{\theta_i \text{Tr}(\bar{\mathbf{Y}}_i X)}_{\text{Oligopoly}} + \underbrace{(\lambda_{c\mathcal{G}(i)} \theta_i K_i - \lambda_{d\mathcal{G}(i)} \theta_i D_i) \text{Tr}(\bar{\mathbf{Y}}_i X_\theta)}_{\text{Direct}} \\ & - 2\theta_i \underbrace{\sum_{j=1}^N (\lambda_{c\mathcal{G}(j)} \theta_j K_j - \lambda_{d\mathcal{G}(j)} \theta_j D_j) [\mathbf{V}_\theta]_{ij} (\bar{\mathbf{v}}_i^\top X_\theta \bar{\mathbf{v}}_j)}_{\text{Product Network Spillover}}. \end{aligned}$$

D Microfoundation

This section develops a microfoundation for the source-dependent managerial risk problem. I embed a principal-agent framework into my environment, wherein the representative consumer delegates the operation of firms to managers. Managers receive performance-based compensation contracts tied to realized firm outcomes. Given these contracts, risk-averse managers simultaneously set prices and exert unobservable risk-management effort to maximize their expected utility.

This section proceeds as follows. I first formalize the manager’s problem and derive their optimal effort and pricing decisions. I then demonstrate that this microfounded setup maps exactly into the reduced-form specification analyzed earlier. Next, I outline the principal’s objective and characterize the optimal contract. Finally, I solve for the decentralized equilibrium and contrast it with the first-best allocation.

D.1 Managers

The representative consumer acts as the principal but lacks the specialized skills required to operate a firm. Consequently, the principal delegates firm management to agents. I assume each firm is run by exactly one agent, and I refer to the agent operating firm i as manager i .

Linear Contract The principal offers manager i a linear compensation contract tied to firm performance. Specifically, the contract consists of two components. The first component provides a baseline salary and a share of the firm’s expected profit. The second component adjusts compensation based on realized revenues and the cost of goods sold.⁹ Formally, the compensation W_{it} received by manager i at time t is given by

$$W_{it} = \alpha_i + \beta_i \mathbb{E}_t[\pi_{it}] + \delta_{ci}(-\bar{q}_{it}\tilde{c}_{it}) + \delta_{di}\bar{m}_{it}\tilde{q}_{it},$$

where α_i represents the fixed baseline salary and β_i determines the manager’s share of expected profit. The subsequent terms tie the compensation to shocks. The parameters δ_{ci} and δ_{di} capture the sensitivity of the managerial payout to unexpected total costs $\bar{q}_{it}\tilde{c}_{it}$ and unexpected profit variations $\bar{m}_{it}\tilde{q}_{it}$, respectively.

⁹Because profit is the difference between revenue and the cost of goods sold, this specification is mathematically equivalent to a contract written directly on realized profit and realized revenue. The explicit inclusion of expected profit reflects the standard corporate practice of benchmarking executive compensation against management earnings guidance.

Risk Management and Effort Costs In addition to setting prices and quantities, manager i can exert unobservable efforts e_{ci} and e_{di} to reduce the volatility of cost and demand shocks.¹⁰ In practice, managing cost risk involves actively screening for more reliable suppliers. Similarly, mitigating demand risk requires diversifying the customer base to avoid reliance on any single buyer.

I capture the cost of risk management efforts through the following effort cost function

$$C_i(\bar{q}_{it}, \bar{m}_{it}, e_{ci}, e_{di}) = k_{ci}\bar{q}_{it}^2 e_{ci}^\xi + k_{di}\bar{m}_{it}^2 e_{di}^\xi, \quad \text{for } \xi > 1,$$

where ξ governs the effort cost elasticity. The scale parameters k_{ci} and k_{di} capture the baseline difficulty of managing supply and demand shocks, respectively. The first component governs the cost of mitigating supply-side risk and scales with the square of expected output $\bar{q}_{it}^2$. This specification captures the reality that larger operations involve complex supply networks. Consequently, shielding a production operation from supply shocks requires disproportionately more managerial efforts. The second component captures the cost of stabilizing demand and scales with the square of the profit margin $\bar{m}_{it}^2$. Economically, products that command high margins are typically highly differentiated and rely on specialized, narrower consumer segments. Therefore, demand stabilization is more costly for firms extracting high margins than for low-margin firms.

Managerial effort mitigates the volatility of both supply and demand shocks. Let ς_{ci}^2 and ς_{qi}^2 denote the respective baseline variances in the absence of risk management. I formalize the technology of volatility reduction by assuming the post-effort variances take the form

$$\sigma_{ci}^2(e_{ci}) = \frac{\varsigma_{ci}^2}{e_{ci}^\rho}, \quad \sigma_{qi}^2(e_{di}) = \frac{\varsigma_{qi}^2}{e_{di}^\rho},$$

where $\rho > 0$ captures the effort elasticity of variance reduction.¹¹

¹⁰While these effort levels could theoretically vary over time, I show below that time-independent contract parameters induce constant effort choices. I therefore omit time subscripts.

¹¹Note that firm i 's demand shock arises from the exposure to the fundamental shocks to the demand shifter $\tilde{\mathbf{b}}_t$ in the following way

$$\tilde{q}_{it} = \Gamma_i(\tilde{\mathbf{b}}_t).$$

Hence, the post-effort volatility of $\tilde{\mathbf{b}}_t$ has to be consistent with firm i 's volatility

$$\sigma_{qi}^2 = \text{Var}_t(\Gamma_i \tilde{\mathbf{b}}_t) = \Gamma_{ii}^2 \sigma_{bi}^2 + \sum_{j \neq i} \Gamma_{ij}^2 \sigma_{bj}^2, \quad \forall i = 1, \dots, N,$$

where σ_{bi}^2 is the post-effort variances of $\tilde{b}_{it}$.

Firm Objective I assume that managers have CARA preferences over their compensation net effort costs. The utility function of manager i is given by

$$- \exp \left\{ - \eta_i \left[W_{it} - C_i \left(\bar{q}_{it}, \bar{m}_{it}, e_{ci}, e_{di} \right) \right] \right\},$$

where η_i is the absolute risk aversion of manager i .

I now formulate formally the manager problem. Firms compete in prices in a Bertrand fashion. The information set available to managers remains identical to the baseline setting, meaning that they perfectly observe the expected demand vector $\bar{\mathbf{b}}_t$ alongside their own expected marginal cost $\bar{c}_{it}$. The manager simultaneously selects prices, expected quantities, and risk management efforts to maximize expected utility subject to the linear residual demand system. In line with the standard Bertrand framework, managers take the pricing decisions of all competing firms as given. The problem of manager i is

$$\begin{aligned} \max_{p_{it}, \bar{q}_{it}, e_{ci}, e_{di}} \quad & - \exp \left\{ - \eta_i \left[W_{it} - C_i \left(\bar{q}_{it}, \bar{m}_{it}, e_{ci}, e_{di} \right) \right] \right\}, \\ \text{s.t.} \quad & \bar{q}_{it} = \Gamma_i \left(\bar{\mathbf{b}}_t - \mathbf{p}_t \right), \\ & \sigma_{ci}^2(e_{ci}) = \frac{\varsigma_{ci}^2}{e_{ci}^\rho}, \quad \sigma_{di}^2(e_{di}) = \frac{\varsigma_{di}^2}{e_{di}^\rho}, \\ & \text{given } \mathbf{p}_{-it}, \bar{\mathbf{b}}_t \text{ and } \bar{c}_{it}. \end{aligned}$$

Exploiting the assumption that the fundamental shocks are independent and normally distributed, the expected utility maximization problem under CARA preferences simplifies to maximizing the certainty equivalent

$$CE_{it} = \alpha_i + \beta_i \bar{m}_{it} \bar{q}_{it} - k_{ci} \bar{q}_{it}^2 e_{ci}^\xi - k_{di} \bar{m}_{it}^2 e_{di}^\xi - \frac{\eta_i}{2} \left[\delta_{ci}^2 \bar{q}_{it}^2 \frac{\varsigma_{ci}^2}{e_{ci}^\rho} + \delta_{di}^2 \bar{m}_{it}^2 \frac{\varsigma_{di}^2}{e_{di}^\rho} \right].$$

Optimal Efforts The manager's problem is to simultaneously select price (p_{it}) and effort (e_{ci}, e_{di}) to maximize their Certainty Equivalent (CE_{it}). Solving for the optimal efforts yields

$$e_{ci}^* = \left(\frac{\rho \eta_i \delta_{ci}^2 \varsigma_{ci}^2}{2 \xi k_{ci}} \right)^{\frac{1}{\xi + \rho}}, \quad e_{di}^* = \left(\frac{\rho \eta_i \delta_{di}^2 \varsigma_{di}^2}{2 \xi k_{di}} \right)^{\frac{1}{\xi + \rho}}. \quad (2)$$

As noted earlier, the optimal effort choices are time invariant. Furthermore, these optimal effort are strictly increasing in the baseline variances of the shocks as managers are risk averse. The optimal effort functions depend on the sensitivities to unexpected costs and profits. This dependence provides the principal with the levers to induce the desired levels of managerial effort through the compensation contract.

At this optimum, the ratio of the manager's required premium to their physical effort cost is strictly fixed by the parameters ξ and ρ . Specifically, the premium is exactly $\frac{\xi}{\rho}$ times the effort cost.

Equivalence to the Reduced-Form I now establish that the microfounded manager problem is equivalent to the reduced form specification analyzed earlier. Observing that expected profit is given by $\mathbb{E}_t[\pi_{it}] = \bar{m}_{it}\bar{q}_{it}$, and that the conditional profit variances driven by cost and demand shocks are $\text{Var}_t^{(c)}(\pi_{it}) = \bar{q}_{it}^2\sigma_{ci}^2$ and $\text{Var}_t^{(d)}(\pi_{it}) = \bar{m}_{it}^2\sigma_{qi}^2$ respectively, I can rewrite the manager's optimal pricing problem exactly as

$$\begin{aligned} \max_{p_{it}, \bar{q}_{it}} \quad & \mathbb{E}_t[\pi_{it}] - \frac{\lambda_{ci}}{2} \text{Var}_t^{(c)}(\pi_{it}) - \frac{\lambda_{di}}{2} \text{Var}_t^{(d)}(\pi_{it}), \\ \text{s.t.} \quad & \bar{q}_{it} = \Gamma_i(\bar{\mathbf{b}}_t - \mathbf{p}_t), \\ & \text{given } \mathbf{p}_{-it}, \bar{\mathbf{b}}_t \text{ and } \bar{c}_{it}, \end{aligned}$$

where the penalties are defined as $\lambda_{ci} \equiv \frac{\xi + \rho}{\xi} \frac{\eta_i \delta_{ci}^2}{\beta_i}$ and $\lambda_{di} \equiv \frac{\xi + \rho}{\xi} \frac{\eta_i \delta_{di}^2}{\beta_i}$.

Crucially, these penalty parameters are governed entirely by primitives and the contract coefficients. Because the risk penalties are invariant to equilibrium prices and quantities, all characterizations and equilibrium expressions derived in the preceding reduced-form analysis remain valid under this microfoundation.

D.2 The Principal

Having characterized the optimal behavior of managers for a given compensation structure, I now turn to the design of the optimal contract. The principal selects the contract parameters $\{\alpha_i, \beta_i, \delta_{ci}, \delta_{di}\}$ for all $i = 1, \dots, N$ to maximize their objective, subject to the incentive compatibility (IC) and individual rationality (IR) constraints of the managers. In what follows, I first outline the constraints, then construct the principal's objective and finally formalize the complete principal problem.

IC and IR Constraints The feasible contract space is restricted by the managers' willingness to participate and their optimal decisions.

The IC constraints encompass two distinct dimensions of the manager's decision problem. First, the unobservable risk management efforts must satisfy the optimality conditions previously characterized in (2). Second, the manager's optimal pricing strategy imposes a restriction on the expected equilibrium margins and quantities, which must satisfy the

optimal pricing condition

$$\bar{m}_{it} = \Delta_i \bar{q}_{it}^*. \quad (3)$$

Finally, the IR constraint ensures that each manager weakly prefers to accept the contract. Normalizing the outside option to zero, this requires the manager's certainty equivalent to be non-negative, yielding

$$CE_{it} \geq 0. \quad (4)$$

Principal Objective I assume the principal designs a time-invariant contract as compensation contracts are sticky. I therefore evaluate the contracting problem at the stationary mean of the underlying shocks. Accordingly, I suppress the time subscript t in what follows. Recall that the ex-post utility of the representative consumer is given by

$$U(\mathbf{q}) = (\mathbf{b} - \mathbf{c})^\top \mathbf{q} - \frac{1}{2} \mathbf{q}^\top \Gamma^{-1} \mathbf{q}.$$

In my preceding analysis, the consumer chose quantities after the realization of shocks, rendering their risk aversion irrelevant. In this contracting stage, however, the principal must design mechanisms ex ante. I therefore bring the consumer's risk preferences back to the forefront.

The consumer has mean-variance preferences over the utility U . To maintain analytical tractability, I utilize a first-order approximation of the utility variance.¹² Formally, the principal preference is

$$\mathbb{E}[U] - \frac{\eta_p}{2} \text{Var}[dU],$$

where η_p is the absolute risk aversion of the principal and $\text{Var}[dU]$ represents the first-order approximation of the utility variance

$$\text{Var}[dU] = \text{Var}[(\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \Gamma \tilde{\mathbf{b}} - \bar{\mathbf{q}}^\top \tilde{\mathbf{c}}].$$

As this paper focuses on the role of idiosyncratic risks, I restrict the principal from using the contract to correct oligopoly market power. To formalize this restriction and isolate the provision of risk-management incentives, I assume the principal maximizes a pseudo-surplus function. Borrowing from the potential games literature, this pseudo-surplus modifies

¹²This mirrors my formulation of the firm's problem, which likewise discards higher-order variance terms.

the objective by incorporating an adjustment term, $-\frac{1}{2}\bar{\mathbf{q}}^\top \Theta \bar{\mathbf{q}}$, which aligns the principal's baseline objective over expected quantities with the decentralized pricing equilibrium. This formulation ensures that the principal problem remains focused on inducing optimal risk-management efforts, rather than attempting to undo firm market power.

The principal solves

$$\begin{aligned} \max_{\{\alpha_i, \beta_i, \delta_{ci}, \delta_{di}\}_{i=1}^N} \quad & \mathbb{E} \left[U - \sum_{i=1}^N W_i \right] - \frac{\eta_p}{2} \text{Var} [dU] - \frac{1}{2} \bar{\mathbf{q}}^\top \Theta \bar{\mathbf{q}}, \\ \text{s.t.} \quad & \text{IC constraints (2) and (3),} \\ & \text{IR constraints (4).} \end{aligned}$$

In this formulation, the principal selects the optimal contract parameters to maximize the pseudo-surplus net of managerial compensation, subject to the IC and IR constraints.

D.3 Optimal Contract

In this section, I solve for the optimal linear contract. The solution allows me to interpret the economic meaning of the source-dependent risk penalties, λ_{ci} and λ_{di} . I demonstrate that the determinants of λ directly reflect the principal risk aversion, the elasticity of risk management effort, and the social cost of volatility.

This section proceeds as follows. First, the existence of baseline compensation α implies that the IR constraints always bind. Second, I show that the sensitivities to idiosyncratic risk are driven by the principal's risk preferences combined with an inflated cost of effort. I then characterize the optimal profit share and equilibrium quantities. Lastly, I interpret the resulting risk penalties λ_{ci} and λ_{di} .

Optimal α As the principal can freely adjust α_i , the manager's individual rationality constraint always binds. By substituting the physical cost of effort into the expected compensation, I can express the total compensation simply as the physical cost inflated by the hidden effort premium. This simplifies the principal's problem conveniently by removing any direct dependency on β_i ,¹³ yielding

$$\mathbb{E}[W_i] = \left(1 + \frac{\xi}{\rho}\right) [k_{ci} \bar{q}_i^2 (e_{ci}^*)^\xi + k_{di} \bar{m}_i^2 (e_{di}^*)^\xi].$$

Note that the optimal efforts e_{ci}^* and e_{di}^* only depend on δ_{ci} and δ_{di} , respectively.

¹³The β_i still plays a role here as the quantity and margin depends on it.

Optimal δ Using the binding IR constraints, the principal objective can be simplified as

$$U^0(\bar{\mathbf{q}}) = \sum_{i=1}^N \underbrace{\bar{q}_i^2 \left[k_{ci} \left(1 + \frac{\xi}{\rho} \right) (e_{ci}^*)^\xi + \frac{\eta_p \varsigma_{ci}^2}{2(e_{ci}^*)^\rho} \right]}_{\equiv \Psi_{ci}} \\ - \sum_{i=1}^N \underbrace{\left[\bar{m}_i^2 k_{di} \left(1 + \frac{\xi}{\rho} \right) (e_{di}^*)^\xi + \frac{(\eta_p \nu_{di} - \zeta_i) \varsigma_{qi}^2}{2(e_{di}^*)^\rho} \right]}_{\equiv \Psi_{di}},$$

where $U^0(\bar{\mathbf{q}}) = (\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \bar{\mathbf{q}} - \frac{1}{2} \bar{\mathbf{q}}^\top (\Gamma^{-1} + \Theta) \bar{\mathbf{q}}$ is a quadratic function of $\bar{\mathbf{q}}$, Ψ_{ci} is firm i cost-risk utility cost and Ψ_{di} is firm i demand-risk utility cost.

A key technical challenge is whether adjusting the risk sensitivities δ_{ci} and δ_{di} alters the equilibrium objects ($\bar{q}_i$, and $\bar{m}_i$). I perform a change of choice variables so that the expected quantity vector $\bar{\mathbf{q}}$ and markup vector $\bar{\mathbf{m}}$ do not depend on δ . I then verify that this change of choice variables is valid, demonstrating that the profit share parameters β_i alone can perfectly recover the same optimal quantities.

Relying on this decoupling, the principal's problem simplifies drastically. I can minimize the total cost-risk and demand-risk utility costs independently over e_{ci} and e_{di} for each firm. To maintain a tractable quadratic structure for the demand-risk utility cost, I factor out the expected markup and approximate it around the risk-free oligopoly markup $(\bar{m}_i^\theta)^2$. This yields constant utility cost coefficients defined as

$$\omega_{ci} \equiv 2 \times \min_{e_{ci}} \left[k_{ci} \left(1 + \frac{\xi}{\rho} \right) e_{ci}^\xi + \frac{\eta_p \varsigma_{ci}^2}{2e_{ci}^\rho} \right], \\ \omega_{di} \equiv 2 \times \min_{e_{di}} \left[k_{di} \left(1 + \frac{\xi}{\rho} \right) e_{di}^\xi + \frac{(\eta_p \nu_{di} - \zeta_i) \varsigma_{qi}^2}{2(\bar{m}_i^\theta)^2 e_{di}^\rho} \right].$$

With these, I can express the risk utility costs as

$$\Psi_{ci}^* = \frac{1}{2} \omega_{ci} \bar{q}_i^2, \quad \Psi_{di}^* = \frac{1}{2} \omega_{di} \bar{m}_i^2.$$

Substituting this back to the principal objective, I note that the problem now depends only on $\bar{\mathbf{q}}$ and $\bar{\mathbf{m}}$.

Optimal β Defining the diagonal agency friction matrices $\Omega_c = \text{diag}(\omega_{c1}, \dots, \omega_{cN})$ and $\Omega_d = \text{diag}(\omega_{d1}, \dots, \omega_{dN})$, and utilizing the inverse demand system $\bar{\mathbf{m}} = \bar{\mathbf{b}} - \bar{\mathbf{c}} - \Gamma^{-1} \bar{\mathbf{q}}$, I can

express the relaxed principal problem over expected quantities as

$$\max_{\bar{\mathbf{q}}} S(\bar{\mathbf{q}}) = (\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \bar{\mathbf{q}} - \frac{1}{2} \bar{\mathbf{q}}^\top (\Gamma^{-1} + \Theta + \Omega_c) \bar{\mathbf{q}} - \frac{1}{2} (\bar{\mathbf{b}} - \bar{\mathbf{c}} - \Gamma^{-1} \bar{\mathbf{q}})^\top \Omega_d (\bar{\mathbf{b}} - \bar{\mathbf{c}} - \Gamma^{-1} \bar{\mathbf{q}}).$$

Applying the same approximation utilized in the earlier section and solving this maximization problem yields the principal's optimal expected quantity vector

$$\bar{\mathbf{q}}^P = (\Gamma^{-1} + \Theta + \Omega_c - \Theta^2 \Omega_d)^{-1} (\bar{\mathbf{b}} - \bar{\mathbf{c}}).$$

This structural form perfectly mirrors the aggregate equilibrium quantity $\bar{\mathbf{q}} = (\Gamma^{-1} + \Delta)^{-1} (\bar{\mathbf{b}} - \bar{\mathbf{c}})$ derived in my earlier section. It remains to verify my conjecture that this target allocation can be implemented solely by adjusting the profit-sharing parameter. I confirm this guess and prove that the principal strictly implements these quantities by setting

$$\beta_i = \frac{\rho}{\rho + \xi}.$$

Risk Penalties λ I now provide an economic interpretation of the reduced-form risk penalties through the lens of this microfoundation. Substituting the optimal contract parameters δ_{ci} and β_i into the expression for λ_{ci} yields the cost-risk penalty

$$\lambda_{ci} = \frac{\xi + \rho}{\xi} \eta_p.$$

This cost-risk penalty explicitly isolates the risk aversion of the principal, η_p , amplified by the agency friction. Because risk-management effort is unobservable, the principal must pay an incentive premium to the manager. This agency cost inflates the effective risk penalty by the multiplier $\frac{\xi + \rho}{\xi}$.

I next evaluate the demand-risk penalty, which takes the form

$$\lambda_{di} = \frac{\xi + \rho}{\xi} \frac{\eta_p \nu_{di} - \zeta_i}{(\bar{m}_i^\theta)^2}.$$

The economic forces driving this parameter are more nuanced. The term $\eta_p \nu_{di}$ reflects the principal's aversion to demand volatility. The coefficient ν_{di} accounts for the structure of the product network, recognizing that the realized quantities $\tilde{q}_i$ are correlated even when the underlying preference shocks $\tilde{b}_i$ are independent.

The term ζ_i captures an option value in demand volatility. Because the representative consumer allocates expenditure after the realization of shocks, they optimally substitute away from products with negative shocks and increase consumption of products with positive

shocks. Consequently, a risk-neutral principal actually benefits from idiosyncratic demand volatility. The numerator $\eta_p \nu_{di} - \zeta_i$ perfectly captures the net balance between the principal's risk aversion and this substitution-driven option value. This is once again inflated by the agency multiplier $\frac{\xi + \rho}{\xi}$.

E Welfare

In this section, I study the welfare implications of idiosyncratic risks using the principal-agent microfoundation developed in the previous section. I proceed by first characterizing the first-best allocations. I then define the aggregate deadweight loss and decompose it into inefficiencies driven by oligopoly market power and agency frictions.

E.1 First-Best

In an economy without information frictions, a central planner perfectly observes manager efforts and dictates the effort levels that minimize the sum of utility effort costs and the principal's risk aversion. Furthermore, the planner directly chooses the expected quantities produced by each firm.

First-Best Efforts Given the physical effort cost $C_i = k_{ci} \bar{q}_i^2 e_{ci}^\xi + k_{di} \bar{m}_i^2 e_{di}^\xi$, the planner's optimal effort choices are decoupled across firms and risk types. I define the first-best utility cost coefficients analogously to the second-best. The planner minimizes the identical utility costs but avoids paying the manager the incentive premium. Approximating the demand-risk frictions around the risk-free oligopoly markup as before, the first-best coefficients are given by

$$\begin{aligned}\omega_{ci}^{FB} &\equiv 2 \times \min_{e_{ci}} \left[k_{ci} e_{ci}^\xi + \frac{\eta_p \varsigma_{ci}^2}{2 e_{ci}^\rho} \right], \\ \omega_{di}^{FB} &\equiv 2 \times \min_{e_{di}} \left[k_{di} e_{di}^\xi + \frac{(\eta_p \nu_{di} - \zeta_i) \varsigma_{qi}^2}{2 (\bar{m}_i^\theta)^2 e_{di}^\rho} \right].\end{aligned}$$

Defining the diagonal matrices $\Omega_c^{FB} = \text{diag}(\omega_{c1}^{FB}, \dots, \omega_{cN}^{FB})$ and $\Omega_d^{FB} = \text{diag}(\omega_{d1}^{FB}, \dots, \omega_{dN}^{FB})$, I can directly compare this first-best risk utility costs to that of the principal-agent problem.

I show that the first-best and second-best coefficients of risk utility costs are related as

$$\Omega_c = \Upsilon \Omega_c^{FB} \quad \text{and} \quad \Omega_d = \Upsilon \Omega_d^{FB},$$

where the multiplier $\Upsilon \equiv \left(1 + \frac{\xi}{\rho}\right)^{\frac{\rho}{\xi+\rho}}$ is a scalar greater than one. This multiplier isolates the utility cost of hidden action, demonstrating that agency frictions inflate the cost of risk management.

First-Best Quantities Substituting the optimized first-best efforts into the planner's objective yields

$$\max_{\bar{\mathbf{q}}} S(\bar{\mathbf{q}}) = (\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \bar{\mathbf{q}} - \frac{1}{2} \bar{\mathbf{q}}^\top (\Gamma^{-1} + \Omega_c^{FB}) \bar{\mathbf{q}} - \frac{1}{2} (\bar{\mathbf{b}} - \bar{\mathbf{c}} - \Gamma^{-1} \bar{\mathbf{q}})^\top \Omega_d^{FB} (\bar{\mathbf{b}} - \bar{\mathbf{c}} - \Gamma^{-1} \bar{\mathbf{q}}).$$

Applying the same approximation utilized in the decentralized setting, the first-order conditions yield the first-best quantity vector

$$\bar{\mathbf{q}}^{FB} = (\Gamma^{-1} + \Omega_c^{FB})^{-1} (\bar{\mathbf{b}} - \bar{\mathbf{c}}).$$

Comparing this efficient allocation to the decentralized equilibrium highlights three distinct margins of improvement. First, the planner resolves the standard oligopoly friction as reflected in the disappearance of the matrix Θ . Second, the effective cost-risk penalty decreases from Ω_c to Ω_c^{FB} . Because the planner perfectly observes manager effort, they avoid the ξ/ρ incentive premium required under agency friction. This allows them to choose a higher level of cost-risk reduction.

Finally, the demand-shock component vanishes from the first-best allocation. This disappearance stems from the dependence of demand-risk management on the margin. Because the effort utility cost required to stabilize demand scales with the squared margin, and the planner optimally drives this close to zero to achieve efficiency, the cost of mitigating demand risk becomes negligible. The planner can then eliminate demand volatility, leaving the first-best allocation governed solely by consumer preferences and the efficient cost-risk frictions.

E.2 Deadweight Loss

To quantify the welfare losses in the decentralized economy, I evaluate allocations using the surplus function¹⁴

$$S(\Omega_c, \Omega_d, \bar{\mathbf{q}}) = (\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \bar{\mathbf{q}} - \frac{1}{2} \bar{\mathbf{q}}^\top (\Gamma^{-1} + \Omega_c) \bar{\mathbf{q}} - \frac{1}{2} (\bar{\mathbf{b}} - \bar{\mathbf{c}} - \Gamma^{-1} \bar{\mathbf{q}})^\top \Omega_d (\bar{\mathbf{b}} - \bar{\mathbf{c}} - \Gamma^{-1} \bar{\mathbf{q}}).$$

¹⁴While this objective explicitly measures consumer welfare, it is mathematically equivalent to total surplus. Because the managers' individual rationality constraints bind strictly at their zero-normalized outside option, the representative consumer extracts all surplus generated in the economy.

I construct allocations to isolate the impact of each friction. Let superscripts denote the specific friction being eliminated (evaluated at its first-best benchmark). The optimal allocation in an economy with agency frictions but without oligopoly competition is given by

$$\bar{\mathbf{q}}^\Theta = (\Gamma^{-1} + \Omega_c)^{-1}(\bar{\mathbf{b}} - \bar{\mathbf{c}}).$$

Consistent with my earlier intuition, eliminating market power drives the margins to zero, eliminating the demand-risk friction in the process. Conversely, the allocation under oligopoly market power but frictionless risk management is

$$\bar{\mathbf{q}}^{\Omega_c, \Omega_d} = (\Gamma^{-1} + \Theta + \Omega_c^{FB} - \Theta^2 \Omega_d^{FB})^{-1}(\bar{\mathbf{b}} - \bar{\mathbf{c}}).$$

The total deadweight loss (DWL) of the decentralized economy relative to the first-best benchmark is strictly defined as

$$DWL = S(\Omega_c^{FB}, \Omega_d^{FB}, \bar{\mathbf{q}}^{FB}) - S(\Omega_c, \Omega_d, \bar{\mathbf{q}}^*).$$

Because the surplus function is non-linear in the frictions, the marginal welfare gain from eliminating one friction depends heavily on whether the others remain active. To provide a decomposition of these interacting distortions, I adopt a Shapley-value approach, decomposing the efficiency loss as

$$DWL = DWL^\Theta + DWL^{\Omega_c} + DWL^{\Omega_d}.$$

Here, DWL^Θ captures the welfare loss attributable to oligopoly market power, while DWL^{Ω_c} and DWL^{Ω_d} isolate the losses driven by the cost-risk and demand-risk agency frictions, respectively. This decomposition ensures that the individual components sum up to the total deadweight loss. The idea is I iterate through all the possible combinations of frictions and apply the weights according to the number of paths that allows me to proceed. The

exact formulas governing this decomposition are

$$\begin{aligned}
DWL^\Theta &= \frac{1}{3} [S(\Omega_c^{FB}, \Omega_d^{FB}, \bar{\mathbf{q}}^{FB}) - S(\Omega_c^{FB}, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Omega_c, \Omega_d})] \\
&\quad + \frac{1}{6} [S(\Omega_c, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Theta, \Omega_d}) - S(\Omega_c, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Omega_d})] \\
&\quad + \frac{1}{6} [S(\Omega_c^{FB}, \Omega_d, \bar{\mathbf{q}}^{\Theta, \Omega_c}) - S(\Omega_c^{FB}, \Omega_d, \bar{\mathbf{q}}^{\Omega_c})] \\
&\quad + \frac{1}{3} [S(\Omega_c, \Omega_d, \bar{\mathbf{q}}^\Theta) - S(\Omega_c, \Omega_d, \bar{\mathbf{q}})] ,
\end{aligned}$$

$$\begin{aligned}
DWL^{\Omega_c} &= \frac{1}{3} [S(\Omega_c^{FB}, \Omega_d^{FB}, \bar{\mathbf{q}}^{FB}) - S(\Omega_c, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Theta, \Omega_d})] \\
&\quad + \frac{1}{6} [S(\Omega_c^{FB}, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Omega_c, \Omega_d}) - S(\Omega_c, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Omega_d})] \\
&\quad + \frac{1}{6} [S(\Omega_c^{FB}, \Omega_d, \bar{\mathbf{q}}^{\Theta, \Omega_c}) - S(\Omega_c, \Omega_d, \bar{\mathbf{q}}^\Theta)] \\
&\quad + \frac{1}{3} [S(\Omega_c^{FB}, \Omega_d, \bar{\mathbf{q}}^{\Omega_c}) - S(\Omega_c, \Omega_d, \bar{\mathbf{q}})] ,
\end{aligned}$$

$$\begin{aligned}
DWL^{\Omega_d} &= \frac{1}{3} [S(\Omega_c^{FB}, \Omega_d^{FB}, \bar{\mathbf{q}}^{FB}) - S(\Omega_c^{FB}, \Omega_d, \bar{\mathbf{q}}^{\Theta, \Omega_c})] \\
&\quad + \frac{1}{6} [S(\Omega_c^{FB}, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Omega_c, \Omega_d}) - S(\Omega_c^{FB}, \Omega_d, \bar{\mathbf{q}}^{\Omega_c})] \\
&\quad + \frac{1}{6} [S(\Omega_c, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Theta, \Omega_d}) - S(\Omega_c, \Omega_d, \bar{\mathbf{q}}^\Theta)] \\
&\quad + \frac{1}{3} [S(\Omega_c, \Omega_d^{FB}, \bar{\mathbf{q}}^{\Omega_d}) - S(\Omega_c, \Omega_d, \bar{\mathbf{q}})] .
\end{aligned}$$

Bounding the Agency Friction To quantitatively assess the welfare implications of the agency frictions, the critical parameter is the agency multiplier Υ . This multiplier maps directly to the underlying elasticities of risk-management effort. Because direct empirical estimates for these specific risk-management elasticities are scarce in the literature, precisely calibrating Υ poses a challenge.

I overcome this empirical limitation by establishing a theoretical upper bound. The functional form of the structural multiplier guarantees that $\Upsilon \leq e^{1/e} \approx 1.44$. This is convenient for welfare analysis. It allows me to compute a robust upper bound on the maximum deadweight loss attributable to agency frictions, regardless of the exact effort elasticities. In the quantitative exercises that follow, I exploit this bound to evaluate the worst-case deadweight loss induced by the agency friction.

F Microfoundation: Omitted Derivation

F.1 Derivation of Expected Utility

The utility of the representative consumer is

$$U = \mathbf{b}^\top \mathbf{q} - \frac{1}{2} \mathbf{q}^\top \Gamma^{-1} \mathbf{q} - \mathbf{c}^\top \mathbf{q}.$$

The random variables are

$$\mathbf{b} = \bar{\mathbf{b}} + \tilde{\mathbf{b}}, \quad \mathbf{c} = \bar{\mathbf{c}} + \tilde{\mathbf{c}}, \quad \mathbf{q} = \bar{\mathbf{q}} + \tilde{\mathbf{q}}.$$

I focus on Bertrand competition. Hence, the idiosyncratic quantity shocks are not affected by prices but only the idiosyncratic demand shifter $\tilde{\mathbf{b}}$

$$\tilde{\mathbf{q}} = \Gamma \tilde{\mathbf{b}}.$$

My goal here is to compute

$$\mathbb{E}[U] = \mathbb{E} \left[\mathbf{b}^\top \mathbf{q} - \frac{1}{2} \mathbf{q}^\top \Gamma^{-1} \mathbf{q} - \mathbf{c}^\top \mathbf{q} \right].$$

I now take the expectation of each term in the utility individually.

The first term

Evaluating the first term yields

$$\begin{aligned} \mathbb{E}[\mathbf{b}^\top \mathbf{q}] &= \mathbb{E}[(\bar{\mathbf{b}} + \tilde{\mathbf{b}})^\top (\bar{\mathbf{q}} + \tilde{\mathbf{q}})] \\ &= \bar{\mathbf{b}}^\top \bar{\mathbf{q}} + \mathbb{E}[\tilde{\mathbf{b}}^\top \tilde{\mathbf{q}}]. \end{aligned}$$

This relies on $\mathbb{E}[\tilde{\mathbf{b}}] = 0$ and $\mathbb{E}[\tilde{\mathbf{q}}] = 0$. Substituting $\tilde{\mathbf{q}} = \Gamma \tilde{\mathbf{b}}$ yields

$$\mathbb{E}[\mathbf{b}^\top \mathbf{q}] = \bar{\mathbf{b}}^\top \bar{\mathbf{q}} + \mathbb{E}[\tilde{\mathbf{b}}^\top \Gamma \tilde{\mathbf{b}}].$$

The second term

Evaluating the second term yields

$$\begin{aligned} \mathbb{E}[\mathbf{q}^\top \Gamma^{-1} \mathbf{q}] &= \mathbb{E}[(\bar{\mathbf{q}} + \tilde{\mathbf{q}})^\top \Gamma^{-1} (\bar{\mathbf{q}} + \tilde{\mathbf{q}})] \\ &= \bar{\mathbf{q}}^\top \Gamma^{-1} \bar{\mathbf{q}} + \mathbb{E}[\tilde{\mathbf{q}}^\top \Gamma^{-1} \tilde{\mathbf{q}}] \\ &= \bar{\mathbf{q}}^\top \Gamma^{-1} \bar{\mathbf{q}} + \mathbb{E}[\tilde{\mathbf{b}}^\top \Gamma \tilde{\mathbf{b}}]. \end{aligned}$$

Note that I substitute $\tilde{\mathbf{q}} = \Gamma \tilde{\mathbf{b}}$ to reach the final equation.

The third term

Lastly, I compute the total cost term. I utilize the assumption that cost shocks $\tilde{\mathbf{c}}$ and demand shocks $\tilde{\mathbf{b}}$ are independent.

$$\begin{aligned}\mathbb{E}[\mathbf{c}^\top \mathbf{q}] &= \bar{\mathbf{c}}^\top \bar{\mathbf{q}} + \mathbb{E}[\tilde{\mathbf{c}}^\top \tilde{\mathbf{q}}] \\ &= \bar{\mathbf{c}}^\top \bar{\mathbf{q}}\end{aligned}$$

Combining the Terms

Substituting these back to the expected utility gives

$$\begin{aligned}\mathbb{E}[U] &= \left(\bar{\mathbf{b}}^\top \bar{\mathbf{q}} + \mathbb{E}[\tilde{\mathbf{b}}^\top \Gamma \tilde{\mathbf{b}}] \right) - \frac{1}{2} \left(\bar{\mathbf{q}}^\top \Gamma^{-1} \bar{\mathbf{q}} + \mathbb{E}[\tilde{\mathbf{b}}^\top \Gamma \tilde{\mathbf{b}}] \right) - \bar{\mathbf{c}}^\top \bar{\mathbf{q}} \\ &= \underbrace{\bar{\mathbf{b}}^\top \bar{\mathbf{q}} - \frac{1}{2} \bar{\mathbf{q}}^\top \Gamma^{-1} \bar{\mathbf{q}} - \bar{\mathbf{c}}^\top \bar{\mathbf{q}}}_{\equiv U^0} + \frac{1}{2} \mathbb{E}[\tilde{\mathbf{b}}^\top \Gamma \tilde{\mathbf{b}}].\end{aligned}$$

where U^0 is the utility at average. The expected utility is the sum of the utility at average and a volatility term in the idiosyncratic demand shock. The last term represents the convexity in demand shocks.

Computing the convexity term

I now compute the scalar expectation $\mathbb{E}[\tilde{\mathbf{b}}^\top \Gamma \tilde{\mathbf{b}}]$

$$\begin{aligned}\mathbb{E}[\text{Tr}(\tilde{\mathbf{b}}^\top \Gamma \tilde{\mathbf{b}})] &= \mathbb{E}[\text{Tr}(\Gamma \tilde{\mathbf{b}} \tilde{\mathbf{b}}^\top)] \\ &= \text{Tr}(\Gamma \mathbb{E}[\tilde{\mathbf{b}} \tilde{\mathbf{b}}^\top]) \\ &= \text{Tr}(\Gamma \Sigma_b)\end{aligned}$$

Because the preference shock covariance matrix is diagonal, $\Sigma_b = \text{diag}(\sigma_{b1}^2, \dots, \sigma_{bn}^2)$, the trace reduces to the sum of the products of the diagonals

$$\text{Tr}(\Gamma \Sigma_b) = \sum_{j=1}^N \Gamma_{jj} \sigma_{bj}^2$$

I now convert $\{\sigma_{bi}^2\}_{i=1}^N$ to $\{\sigma_{qi}^2\}_{i=1}^N$. Note that

$$\sigma_{qi}^2 = \sum_{j=1}^N \Gamma_{ij}^2 \sigma_{bj}^2$$

Let $\boldsymbol{\sigma}_q^2$ and $\boldsymbol{\sigma}_b^2$ be $N \times 1$ column vectors of these variances. The relationship between them

is

$$\boldsymbol{\sigma}_q^2 = (\Gamma \circ \Gamma) \boldsymbol{\sigma}_b^2 \implies \boldsymbol{\sigma}_b^2 = (\Gamma \circ \Gamma)^{-1} \boldsymbol{\sigma}_q^2.$$

Let $\boldsymbol{\gamma}_d = (\Gamma_{11}, \dots, \Gamma_{NN})^\top$ be the vector of the diagonal elements of Γ . The trace can be rewritten as

$$\begin{aligned} \text{Tr}(\Gamma \Sigma_b) &= \boldsymbol{\gamma}_d^\top \boldsymbol{\sigma}_b^2 \\ &= \boldsymbol{\gamma}_d^\top (\Gamma \circ \Gamma)^{-1} \boldsymbol{\sigma}_q^2 \end{aligned}$$

Define

$$\boldsymbol{\zeta}^\top \equiv \boldsymbol{\gamma}_d^\top (\Gamma \circ \Gamma)^{-1}$$

Let ζ_i be the i -th element of $\boldsymbol{\zeta}$. The convexity term can be written as a sum of network-weighted variances

$$\text{Tr}(\Gamma \Sigma_b) = \sum_{i=1}^N \zeta_i \sigma_{qi}^2$$

Hence,

$$\mathbb{E}[U] = \underbrace{\bar{\mathbf{b}}^\top \bar{\mathbf{q}} - \frac{1}{2} \bar{\mathbf{q}}^\top \Gamma^{-1} \bar{\mathbf{q}} - \bar{\mathbf{c}}^\top \bar{\mathbf{q}}}_{\equiv U^0} + \sum_{i=1}^N \frac{\zeta_i}{2} \sigma_{qi}^2(e_{di})$$

F.2 Derivation of Variance

I now compute the representative consumer first-order variance $\text{Var}[dU]$. I first simplify dU

$$\begin{aligned} dU &= \bar{\mathbf{q}}^\top \tilde{\mathbf{b}} + (\bar{\mathbf{b}} - \Gamma^{-1} \bar{\mathbf{q}} - \bar{\mathbf{c}})^\top \tilde{\mathbf{q}} - \bar{\mathbf{q}}^\top \tilde{\mathbf{c}} \\ &= \bar{\mathbf{q}}^\top \tilde{\mathbf{b}} + (\bar{\mathbf{b}} - \Gamma^{-1} \bar{\mathbf{q}} - \bar{\mathbf{c}})^\top \Gamma \tilde{\mathbf{b}} - \bar{\mathbf{q}}^\top \tilde{\mathbf{c}} \\ &= (\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \Gamma \tilde{\mathbf{b}} - \bar{\mathbf{q}}^\top \tilde{\mathbf{c}}. \end{aligned}$$

Computing the first-order variance yields

$$\begin{aligned}
\text{Var}[dU] &= \text{Var} \left[(\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \Gamma \tilde{\mathbf{b}} - \bar{\mathbf{q}}^\top \tilde{\mathbf{c}} \right] \\
&= (\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \Gamma \Sigma_b \Gamma (\bar{\mathbf{b}} - \bar{\mathbf{c}}) + \sum_{i=1}^N \bar{q}_i^2 \sigma_{ci}^2 \\
&= \text{Tr} \left[(\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \Gamma \Sigma_b \Gamma (\bar{\mathbf{b}} - \bar{\mathbf{c}}) \right] + \sum_{i=1}^N \bar{q}_i^2 \sigma_{ci}^2 \\
&= \text{Tr} \left[\Gamma (\bar{\mathbf{b}} - \bar{\mathbf{c}}) (\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \Gamma \Sigma_b \right] + \sum_{i=1}^N \bar{q}_i^2 \sigma_{ci}^2 \\
&= \boldsymbol{\nu}_d^\top \boldsymbol{\sigma}_q^2 + \sum_{i=1}^N \bar{q}_i^2 \sigma_{ci}^2 \\
&= \sum_{i=1}^N \nu_{di} \sigma_{qi}^2 + \sum_{i=1}^N \bar{q}_i^2 \sigma_{ci}^2,
\end{aligned}$$

where $\boldsymbol{\nu}_d \equiv (\Gamma \circ \Gamma)^{-1} \text{diag} \left(\Gamma (\bar{\mathbf{b}} - \bar{\mathbf{c}}) (\bar{\mathbf{b}} - \bar{\mathbf{c}})^\top \Gamma \right)$, and ν_{di} is the i -th element of $\boldsymbol{\nu}_d$.

F.3 Principal Optimal Quantity

To find the principal optimal expected quantity vector $\bar{\mathbf{q}}^P$, I maximize the principal's expected surplus. Let $\mathbf{v} \equiv \bar{\mathbf{b}} - \bar{\mathbf{c}}$ denote the intrinsic baseline profitability. The unconstrained objective function over quantities is given by

$$S(\bar{\mathbf{q}}) = \mathbf{v}^\top \bar{\mathbf{q}} - \frac{1}{2} \bar{\mathbf{q}}^\top (\Gamma^{-1} + \Theta + \Omega_c) \bar{\mathbf{q}} - \frac{1}{2} (\mathbf{v} - \Gamma^{-1} \bar{\mathbf{q}})^\top \Omega_d (\mathbf{v} - \Gamma^{-1} \bar{\mathbf{q}}).$$

Taking the gradient with respect to the vector $\bar{\mathbf{q}}$ (noting that Γ^{-1} and the diagonal matrix Ω_d are symmetric), the first-order condition is

$$\nabla_{\bar{\mathbf{q}}} S(\bar{\mathbf{q}}) = \mathbf{v} - (\Gamma^{-1} + \Theta + \Omega_c) \bar{\mathbf{q}} + \Gamma^{-1} \Omega_d (\mathbf{v} - \Gamma^{-1} \bar{\mathbf{q}}) = \mathbf{0}.$$

Expanding the terms and isolating $\bar{\mathbf{q}}$ on the left-hand side yields

$$\begin{aligned}
(\Gamma^{-1} + \Theta + \Omega_c + \Gamma^{-1} \Omega_d \Gamma^{-1}) \bar{\mathbf{q}} &= \mathbf{v} + \Gamma^{-1} \Omega_d \mathbf{v} \\
(\Gamma^{-1} + \Theta + \Omega_c + \Gamma^{-1} \Omega_d \Gamma^{-1}) \bar{\mathbf{q}} &= (I + \Gamma^{-1} \Omega_d) \mathbf{v}.
\end{aligned}$$

Multiplying by the inverse of the matrix on the left-hand side provides the exact optimal

quantity vector

$$\bar{\mathbf{q}}^P = (\Gamma^{-1} + \Theta + \Omega_c + \Gamma^{-1}\Omega_d\Gamma^{-1})^{-1} (I + \Gamma^{-1}\Omega_d)(\bar{\mathbf{b}} - \bar{\mathbf{c}}).$$

To evaluate this target analytically, I apply a first-order approximation around $\tilde{\alpha} = 0$ and $\sigma^2 = 0$. I treat the network spillover parameter $\tilde{\alpha}$ and the structural agency friction matrices Ω_c and Ω_d as first-order perturbations. Assuming $\Gamma = \Theta^{-1} + \tilde{\alpha}\Lambda$, I expand its inverse to first order as $\Gamma^{-1} \approx \Theta - \tilde{\alpha}\Theta\Lambda\Theta$. Let $A \equiv 2\Theta$ denote the unperturbed deterministic baseline matrix (evaluated at $\tilde{\alpha} = 0, \sigma^2 = 0$). Substituting the expanded Γ^{-1} into the exact solution and truncating cross-perturbation terms (where Γ^{-1} simplifies to Θ when multiplying the $\mathcal{O}(\sigma^2)$ matrix Ω_d) yields

$$\bar{\mathbf{q}}^P \approx (A - \tilde{\alpha}\Theta\Lambda\Theta + \Omega_c + \Theta\Omega_d\Theta)^{-1} (I + \Theta\Omega_d)\mathbf{v}.$$

Using the standard first-order matrix perturbation expansion, $(A + \Delta)^{-1} \approx A^{-1} - A^{-1}\Delta A^{-1}$, I expand the inverse matrix

$$\bar{\mathbf{q}}^P \approx [A^{-1} - A^{-1}(-\tilde{\alpha}\Theta\Lambda\Theta + \Omega_c + \Theta\Omega_d\Theta)A^{-1}] (I + \Theta\Omega_d)\mathbf{v}.$$

Expanding the product and truncating higher-order risk interactions (i.e., dropping the term containing $\Omega_d\Theta\Omega_d$ which is $\mathcal{O}(\sigma^4)$), I obtain

$$\bar{\mathbf{q}}^P \approx [A^{-1} + A^{-1}\Theta\Omega_d - A^{-1}(-\tilde{\alpha}\Theta\Lambda\Theta + \Omega_c + \Theta\Omega_d\Theta)A^{-1}] \mathbf{v}.$$

To simplify the demand-risk terms, I isolate all terms containing Ω_d . By rewriting $A^{-1}\Theta\Omega_d$ as $A^{-1}\Theta\Omega_d A A^{-1}$, I factor out A^{-1} on both sides

$$A^{-1}\Theta\Omega_d - A^{-1}\Theta\Omega_d\Theta A^{-1} = A^{-1}[\Theta\Omega_d A - \Theta\Omega_d\Theta]A^{-1}.$$

Substituting the baseline matrix $A = 2\Theta$ into the bracket yields an exact simplification of the demand-interaction terms

$$A^{-1}[\Theta\Omega_d(2\Theta) - \Theta\Omega_d\Theta]A^{-1} = A^{-1}[\Theta\Omega_d\Theta]A^{-1}.$$

Substituting this simplified demand-risk term back into the expanded expression along-

side the other perturbations yields

$$\begin{aligned}\bar{\mathbf{q}}^P &\approx [A^{-1} - A^{-1}(-\tilde{\alpha}\Theta\Lambda\Theta + \Omega_c)A^{-1} + A^{-1}(\Theta\Omega_d\Theta)A^{-1}] \mathbf{v} \\ &= [A^{-1} - A^{-1}(-\tilde{\alpha}\Theta\Lambda\Theta + \Omega_c - \Theta\Omega_d\Theta)A^{-1}] \mathbf{v}.\end{aligned}$$

Recognizing this structure as the exact first-order Taylor expansion for $(A + \Delta')^{-1}$ with the consolidated friction matrix $\Delta' \equiv -\tilde{\alpha}\Theta\Lambda\Theta + \Omega_c - \Theta\Omega_d\Theta$, I re-substitute $\Gamma^{-1} \approx \Theta - \tilde{\alpha}\Theta\Lambda\Theta$ and $\mathbf{v} = \bar{\mathbf{b}} - \bar{\mathbf{c}}$. This gives the final compact inverse form

$$\bar{\mathbf{q}}^P \approx (\Gamma^{-1} + \Theta + \Omega_c - \Theta\Omega_d\Theta)^{-1} (\bar{\mathbf{b}} - \bar{\mathbf{c}}).$$

This shows that, up to a first-order approximation, the optimal quantity vector is governed by a perturbed diagonal friction matrix, preserving the tractability of decentralized implementation.

F.4 Optimal δ, β and Risk Penalties

I now solve for the optimal contract parameters and map them to the reduced form risk penalties. The principal minimizes the cost-risk utility cost Ψ_{ci} and demand-risk utility cost Ψ_{di} with respect to the induced efforts e_{ci} and e_{di} . Since the manager's optimal effort choices are monotone in the contract sensitivities δ_{ci} and δ_{di} , the principal can choose the effort levels by setting the corresponding sensitivities.

For cost risk, the principal minimizes the term in the bracket of Ψ_{ci}

$$\min_{e_{ci}} \left[k_{ci} \left(1 + \frac{\xi}{\rho} \right) e_{ci}^{\xi} + \frac{\eta_p \zeta_{ci}^2}{2e_{ci}^{\rho}} \right].$$

Taking the first order condition with respect to e_{ci} yields the principal's desired effort e_{ci}^*

$$\xi k_{ci} \left(\frac{\xi + \rho}{\rho} \right) (e_{ci}^*)^{\xi-1} - \frac{\rho \eta_p \zeta_{ci}^2}{2(e_{ci}^*)^{\rho+1}} = 0.$$

Rearranging this provides the optimality condition

$$(e_{ci}^*)^{\xi+\rho} = \frac{\rho^2 \eta_p \zeta_{ci}^2}{2\xi(\xi + \rho)k_{ci}}.$$

Equating this desired effort to the manager's optimal effort $e_{ci}^* = \left(\frac{\rho \eta_i \delta_{ci}^2 \zeta_{ci}^2}{2\xi k_{ci}} \right)^{\frac{1}{\xi+\rho}}$ from the

incentive compatibility constraint allows me to solve for the optimal contract sensitivity δ_{ci}

$$\frac{\rho\eta_i\delta_{ci}^2\zeta_{ci}^2}{2\xi k_{ci}} = \frac{\rho^2\eta_p\zeta_{ci}^2}{2\xi(\xi + \rho)k_{ci}}$$

$$\delta_{ci}^2 = \frac{\rho}{\xi + \rho} \frac{\eta_p}{\eta_i}.$$

Applying the symmetric procedure to the demand-risk utility cost Ψ_{di} yields the principal's optimal demand sensitivity δ_{di}

$$\delta_{di}^2 = \frac{\rho}{\xi + \rho} \frac{\eta_p\nu_{di} - \zeta_i}{\eta_i(\bar{m}_i^\theta)^2}.$$

The principal targets the quantity allocation $\bar{\mathbf{q}}^P = (\Gamma^{-1} + \Theta + \Omega_c - \Theta^2\Omega_d)^{-1}(\bar{\mathbf{b}} - \bar{\mathbf{c}})$. To implement this in the decentralized equilibrium where $\bar{\mathbf{q}} = (\Gamma^{-1} + \Delta)^{-1}(\bar{\mathbf{b}} - \bar{\mathbf{c}})$, the principal must align the decentralized risk-adjusted conduct parameter with the efficient wedge. This requires setting $\Delta_i = \theta_i + \omega_{ci} - \theta_i^2\omega_{di}$. Recall the decentralized wedge is $\Delta_i = \theta_i + \lambda_{ci}\sigma_{ci}^2 - \lambda_{di}\theta_i^2\sigma_{qi}^2$. Thus, the contract must induce $\lambda_{ci}\sigma_{ci}^2 = \omega_{ci}$ and $\lambda_{di}\sigma_{qi}^2 = \omega_{di}$. I evaluate the cost-risk parameter ω_{ci} at the optimal effort e_{ci}^*

$$\omega_{ci} = 2 \left[k_{ci} \left(\frac{\xi + \rho}{\rho} \right) (e_{ci}^*)^\xi + \frac{\eta_p\zeta_{ci}^2}{2(e_{ci}^*)^\rho} \right].$$

Using the first order condition $k_{ci} \left(\frac{\xi + \rho}{\rho} \right) (e_{ci}^*)^\xi = \frac{\rho\eta_p\zeta_{ci}^2}{2\xi(e_{ci}^*)^\rho}$, I substitute the effort cost out

$$\begin{aligned} \omega_{ci} &= 2 \left[\frac{\rho\eta_p\zeta_{ci}^2}{2\xi(e_{ci}^*)^\rho} + \frac{\eta_p\zeta_{ci}^2}{2(e_{ci}^*)^\rho} \right] \\ &= \frac{\xi + \rho}{\xi} \eta_p \frac{\zeta_{ci}^2}{(e_{ci}^*)^\rho} \\ &= \frac{\xi + \rho}{\xi} \eta_p \sigma_{ci}^2. \end{aligned}$$

Matching this with $\lambda_{ci}\sigma_{ci}^2 = \omega_{ci}$ implies the target risk penalty is $\lambda_{ci} = \frac{\xi + \rho}{\xi} \eta_p$. Recall from the manager's problem that the reduced form penalty is defined as $\lambda_{ci} \equiv \frac{\xi + \rho}{\xi} \frac{\eta_i\delta_{ci}^2}{\beta_i}$. Substituting my solved optimal δ_{ci}^2 into this definition gives

$$\begin{aligned} \lambda_{ci} &= \frac{\xi + \rho}{\xi} \frac{\eta_i}{\beta_i} \left(\frac{\rho}{\xi + \rho} \frac{\eta_p}{\eta_i} \right) \\ &= \frac{\rho}{\xi\beta_i} \eta_p. \end{aligned}$$

Equating this to the target risk penalty $\frac{\xi+\rho}{\xi}\eta_p$ uniquely identifies the optimal profit sharing parameter

$$\begin{aligned}\frac{\rho}{\xi\beta_i}\eta_p &= \frac{\xi+\rho}{\xi}\eta_p \\ \beta_i &= \frac{\rho}{\xi+\rho}.\end{aligned}$$

This confirms my conjecture. The principal strictly implements the target allocation purely by setting $\beta_i = \frac{\rho}{\xi+\rho}$.

The reduced form risk penalties are thus pinned down by substituting the optimal contract parameters. For cost risk, I established

$$\lambda_{ci} = \frac{\xi+\rho}{\xi}\eta_p.$$

By applying the identical steps to the demand risk friction, the optimal demand risk penalty resolves to

$$\lambda_{di} = \frac{\xi+\rho}{\xi} \frac{\eta_p \nu_{di} - \zeta_i}{(\bar{m}_i^\theta)^2}.$$

G Robustness

In the section, I first provide some evidence that the binning accounts for a substantial portion of cross-sectional variation. Next, I show that my results are robust to alternative binning choices.

G.1 Binning

Table 7 reports the broad Standard Industrial Classification (SIC) sectors used for my baseline partition.

SIC Code Range	Sector Name
100 – 999	Agriculture, Forestry, Fishing
1000 – 1499	Mining
1500 – 1799	Construction
2000 – 3999	Manufacturing
4000 – 4999	Transportation, Comm, Elec, Gas
5000 – 5199	Wholesale Trade
5200 – 5999	Retail Trade
7000 – 8999	Services
9900 – 9999	Nonclassifiable

Table 7: Broad Standard Industrial Classification (SIC) Code.

To demonstrate that my binning strategy accounts for the cross-sectional variation, I conduct a placebo exercise. I randomly sample 30 firms from within a bin and compare them to a placebo group of 30 firms drawn from the full sample. Plotting the revenue time series for both groups reveals that the bin-specific average tracks its firms, leaving little unexplained within-group variation. Visual inspection of Figure 4 confirms that the bin average explains a larger share of the variation among its firms than the placebo average does for the random group. This provides evidence that my binning strategy effectively isolates parts that are driven by common factors.

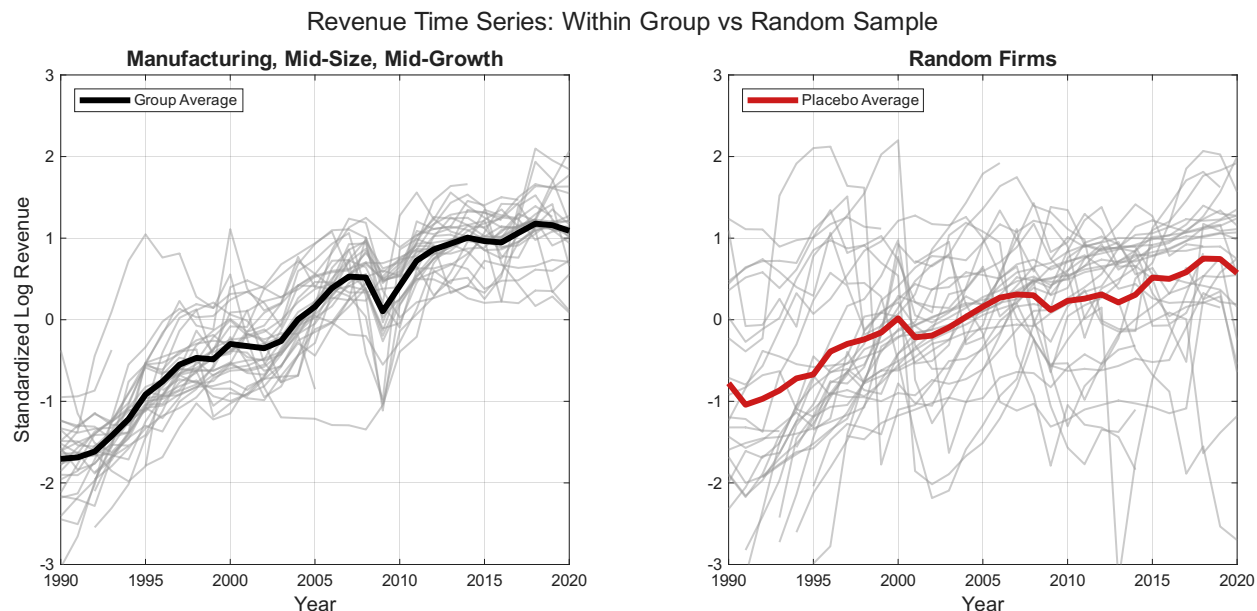


Figure 4: This figure plots the standardize log revenue time series of 30 randomly selected firms from a single bin (manufacturing sector, medium growth, medium revenue) against a placebo group of 30 firms drawn unconditionally from the full sample. The clustering of firms around the bin-specific average demonstrates that the grouping effectively absorbs the relevant cross-sectional variation.

Figure 5 extends this validation exercise across six sectors: mining, manufacturing, transportation, wholesale, retail, and services. For each sector, I plot the revenue time series of 30 randomly sampled firms belonging to the medium-growth, medium-revenue bin. Consistent with my previous example, the tight clustering across all panels confirms that my partitioning strategy accounts for the bulk of the cross-sectional variation.

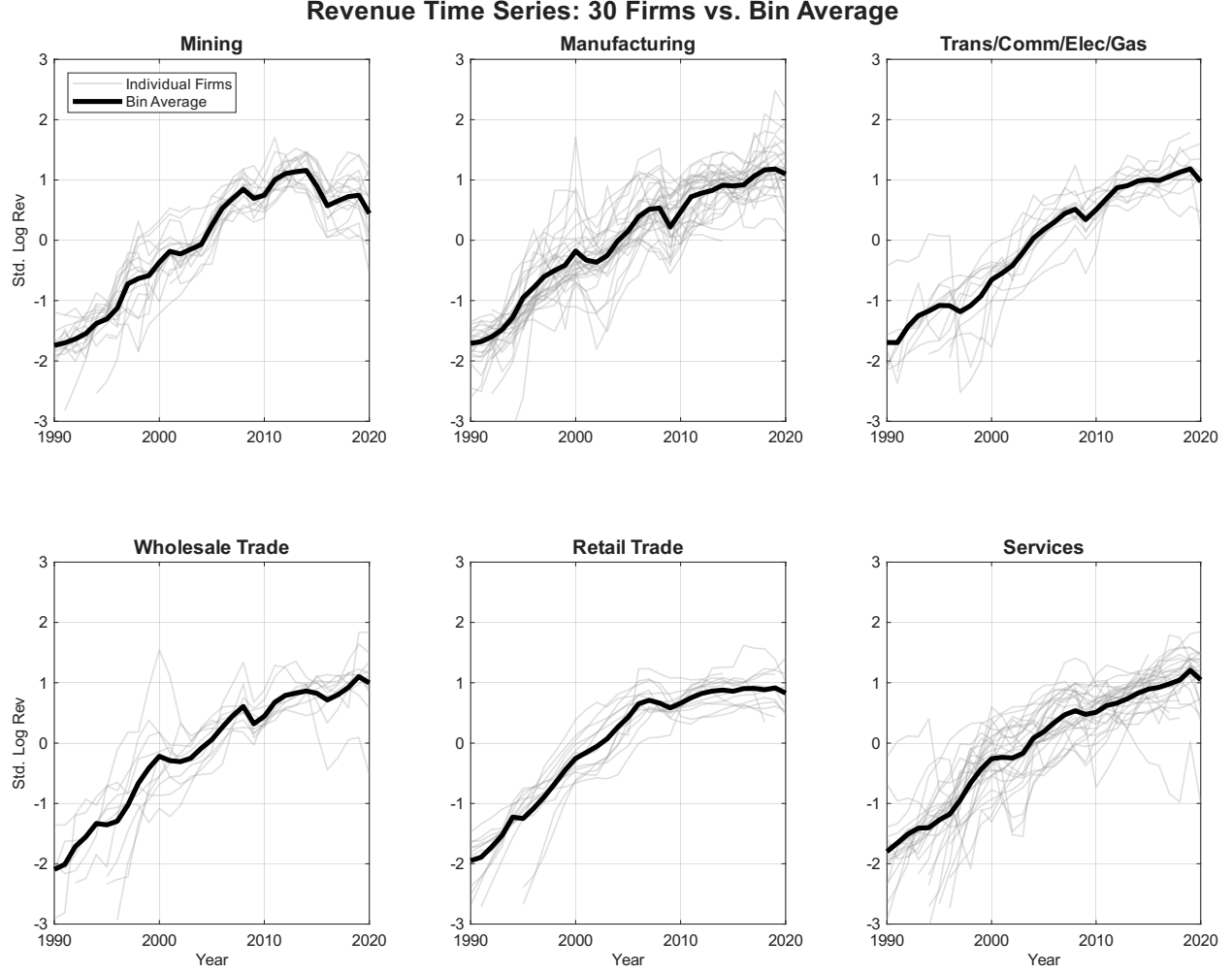


Figure 5: This figure replicates the randomization exercise for six sectors (mining, manufacturing, transportation, wholesale, retail, and services). Each panel plots the revenue time series of 30 randomly drawn firms from the medium-growth, medium-revenue bin within the sector. Across all industries, individual firm cluster around their bin-specific averages, providing evidence that the partition effectively absorbs relevant cross-sectional variation.

G.2 Alternative Binning Choices

To ensure my findings are robust to the choice of groups, I re-estimate the model under alternative binning choices. My baseline specification partitions firms into a 3×3 grid (growth $\times$ revenue) within each broad sector. As increasing the number of bins further is not feasible due to large number of parameters and small number of firms within each bin, I test the sensitivity of my estimates using coarser partitions.

Tables 8 through 10 report the markup decomposition under 2×3 , 3×2 , and 2×2 growth $\times$ revenue grids, respectively. Across all specifications, the magnitude of risk markups are

stable. The risk-neutral baseline continues to account for the vast majority of the net markup.

	Firm Average	Rev-Weighted
Baseline Risk-Neutral	95.90%	94.13%
Cost Risk Premium	2.13%	5.44%
Demand Risk Premium	−0.70%	−0.34%
Risk Market Power	2.67%	0.77%

Table 8: Markup Decomposition (% of Net Markup) with 9 broad sectors, 2 growth $\times$ 3 revenue bins.

	Firm Average	Rev-Weighted
Baseline Risk-Neutral	95.89%	94.21%
Cost Risk Premium	2.14%	5.88%
Demand Risk Premium	−0.73%	−0.87%
Risk Market Power	2.71%	0.78%

Table 9: Markup Decomposition (% of Net Markup) with 9 broad sectors, 3 growth $\times$ 2 revenue bins.

	Firm Average	Rev-Weighted
Baseline Risk-Neutral	93.10%	90.75%
Cost Risk Premium	3.30%	8.32%
Demand Risk Premium	−0.69%	−0.35%
Risk Market Power	4.38%	1.28%

Table 10: Markup Decomposition (% of Net Markup) with 9 broad sectors, 2 growth $\times$ 2 revenue bins.